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# Model inference, some approaches

Learning methods and applications

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Léo Henry

Univ. Rennes, INRIA & CNRS  
Rennes (France)

# Summary

## Passive model learning

First approach

Going further : Cyber-Physical Production Systems

Going further : Software Analysis

## Active learning

Angluin's approach

Adding time

## Take Away

# The Passive Model-Learning Problem

## Definition: Passive learning

**Input**  $(S_+, S_-) \in (\Sigma^*)^{n+m}$

**Output** A model  $\mathcal{A}$  s.t.  $S_+ \subseteq L(\mathcal{A})$  and  $L(\mathcal{A}) \cap S_- = \emptyset$

# The Passive Model-Learning Problem

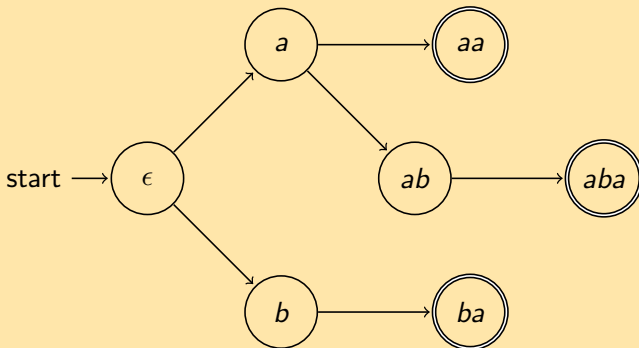
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Consider :  $\mathcal{A} \in \mathcal{DFA}$

Example :  $S_+ = \{aa, ba, aba\}$ ,  $S_- = \{a, ab\}$ .



# Occam's razor

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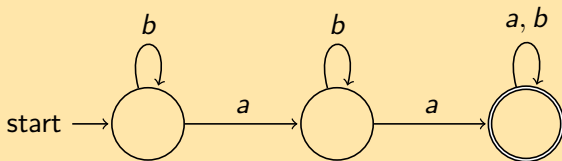
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# Occam's razor

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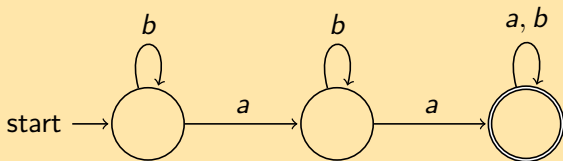
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# Occam's razor

A more general model is required  $\rightsquigarrow$  find a simpler model.

Example :  $S_+ = \{aa, ba, aba\}$ ,  $S_- = \{a, ab\}$ .



## Property: Gold 1978

“Given a sample  $S = (S_+, S_-)$  and  $k \in \mathbb{N}$ , does a DFA with at most  $k$  states and consistent with  $S$  exist?” is NP-complete.

**Input:**  $(S_+, S_-)$

Construct  $PTA(S_+)$

Order the states of  $PTA(S_+) = \{q_0, \dots, q_n\}$  according to the canonical order on words.

**for**  $i \in [1, n]$  **do**

**if**  $q_i$  has not been merged with a smaller state **then**

        Try to merge  $q_i$  with  $q_0, \dots, q_{i-1}$  until a merged DFA does not accept a word in  $S_-$

**end**

**end**

**return** the final DFA

**Algorithm 1:** Regular Positive Negative Inference

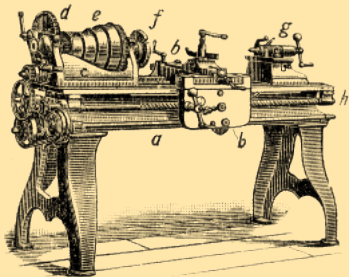


### Property: Oncina and Garcia 1992

Given a sample  $S$ , RPNI provides a consistent DNA in polynomial time.

Furthermore if "enough information" on the original language  $\mathcal{L}$  is provided in  $S$ , then RPNI returns the minimal accepting DFA for  $\mathcal{L}$ .

# Cyber-Physical Production Systems



Lathe, p. 1218.

- ▶ **Complex and critical** systems.



Analysis and verification  
desirable

- ▶  $S_{\text{...}}$  is not accessible

↪ CPPSs are learnt "from text".

## Probabilistic approach

### Property: Gold 1967

Regular languages are not identifiable in the limit from positive examples only.

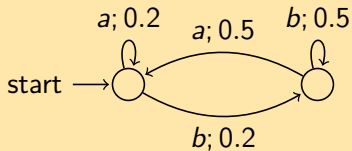
# Probabilistic approach

## Property: Gold 1967

Regular languages are not identifiable in the limit from positive examples only.

## Property: Angluin 1988

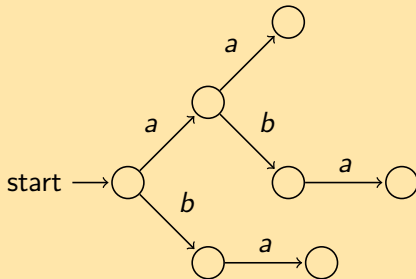
Stochastic regular languages are identifiable from text only with probability 1 (when the probabilities are rational)



# Alergia & MDI

Integrating probabilities

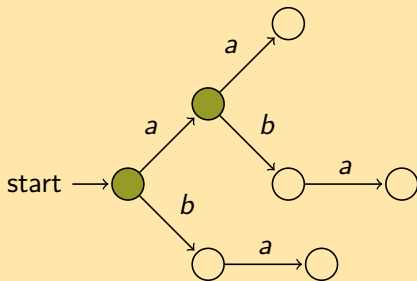
## ALERGIA



# Alergia & MDI

## Integrating probabilities

### ALERGIA



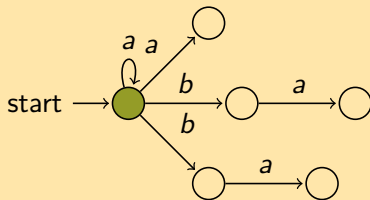
$$\left| \frac{C(q, a)}{C(q)} - \frac{C(q', a)}{C(q')} \right| < K \quad (1)$$

$$\delta(q, a) \text{ and } \delta(q', a) \text{ compatibles} \quad (2)$$

# Alergia & MDI

## Integrating probabilities

### ALERGIA



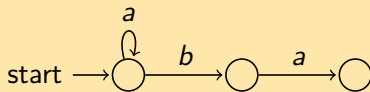
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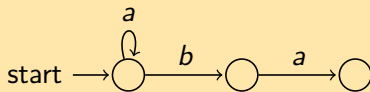
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# Alergia & MDI

Integrating probabilities

## ALERGIA



## MDI

$$\frac{\Delta(A, A')}{|A| - |A'|} \quad (3)$$

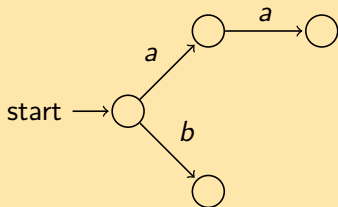
Entropy increase

Generalization

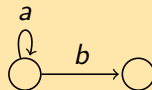
# Adding time

## The splitting operation

General approach :



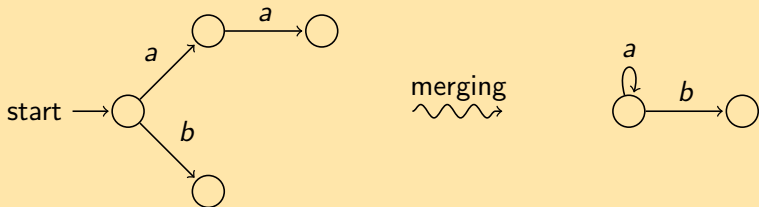
merging  
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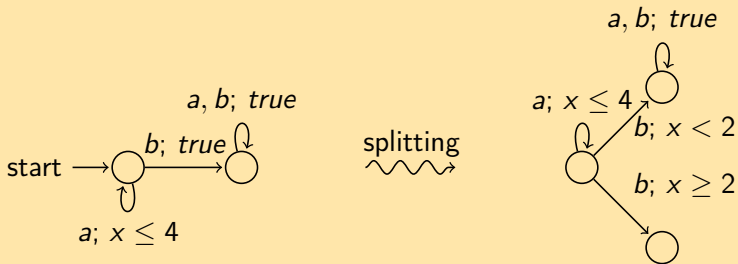
# Adding time

## The splitting operation

General approach :



With timing constraints :



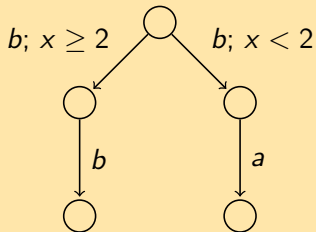
# Algorithms for timed CPPSs



Restricted to **1 clock**, reset at each transition

Subtree difference

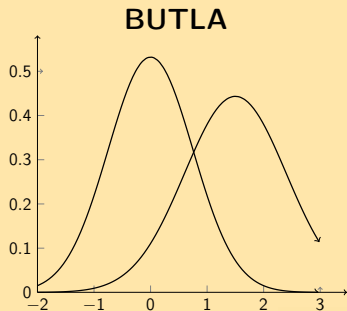
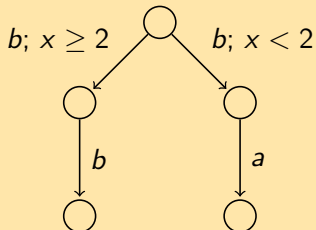
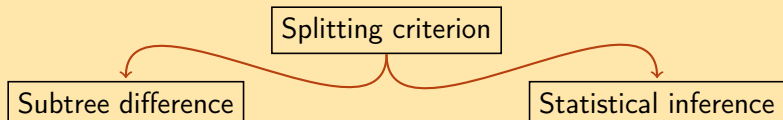
Splitting criterion



# Algorithms for timed CPPSs

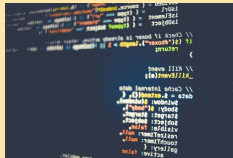


Restricted to **1 clock**, reset at each transition



- ▶ Offline  $\rightsquigarrow$  Online
- ▶ According to a recent survey :
  - A general *learnable* model is required  $\rightarrow$  data-driven modeling ;
  - such a model should be *hybrid* and *explicitly timed* ;
  - a component wise approach is recommended to provide symptoms of the problems.

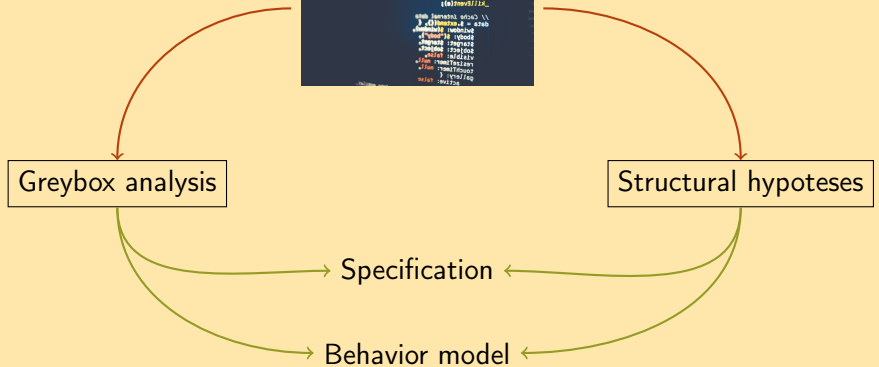
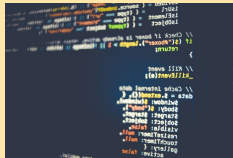
# Software analysis



Greybox analysis

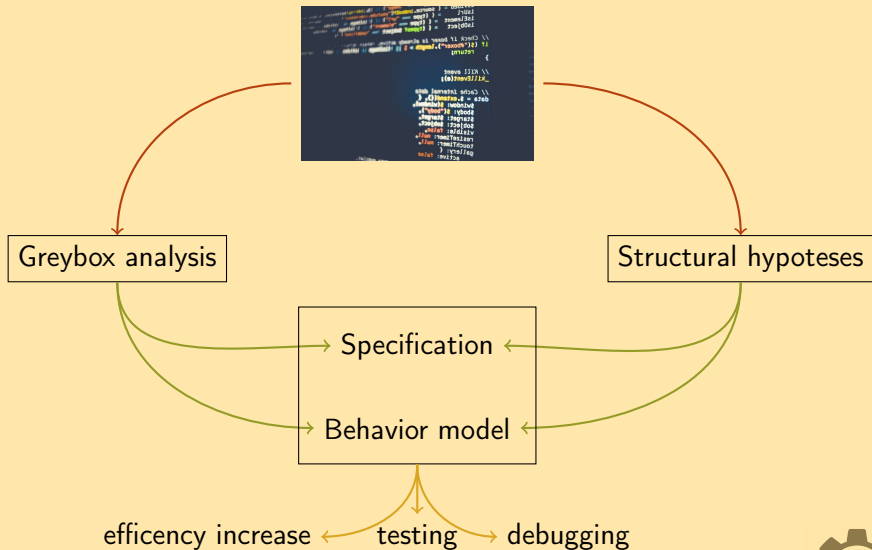
Structural hypotheses

# Software analysis





# Software analysis



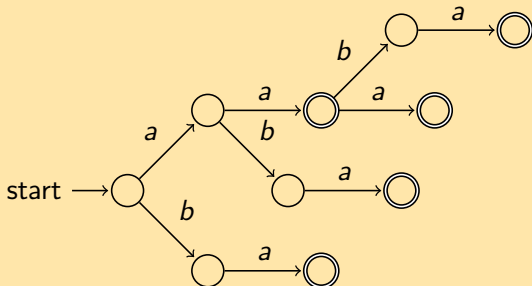
# k-Tail algorithm...

... and its variations



Non stochastic models

**k-Tail** :  $p \sim p' := \forall v \in \Sigma^{\leq k}, p.v \in \mathcal{P} \leftrightarrow p'.v \in \mathcal{P}$



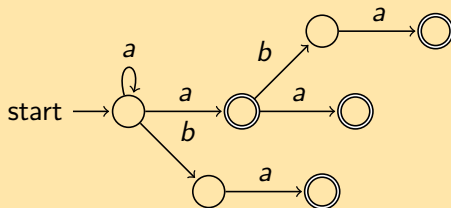
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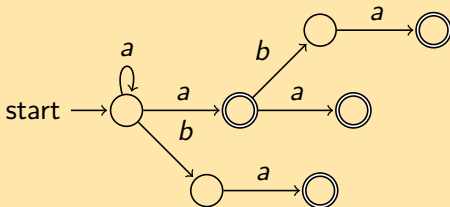
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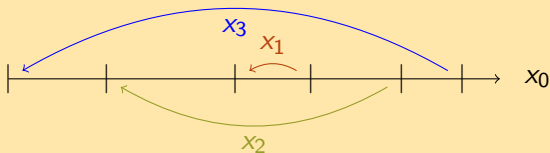
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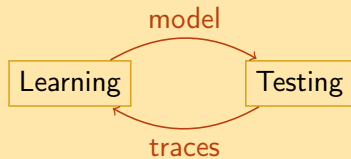
**gk-Tail** : handles parameters by using an ad-hoc function

**Tk-Tail** : adds time modeled by **unbounded** number of clocks.



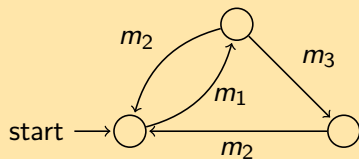
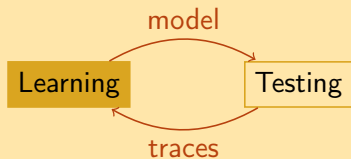
# Integrating testing and learning

... using TAUTOKO



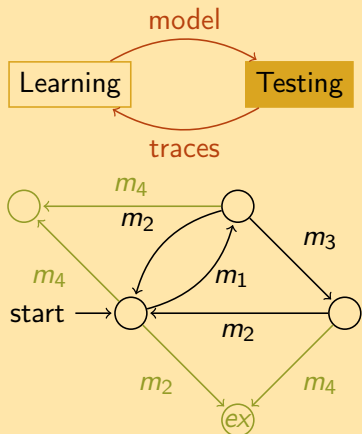
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# Summary

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- First approach

- Going further : Cyber-Physical Production Systems

- Going further : Software Analysis

## Active learning

- Angluin's approach

- Adding time

## Take Away

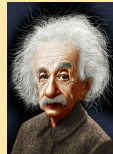


# Active learning

the minimally adequate teacher



learner



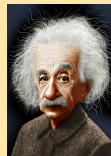
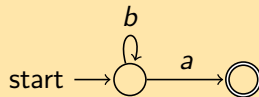
teacher

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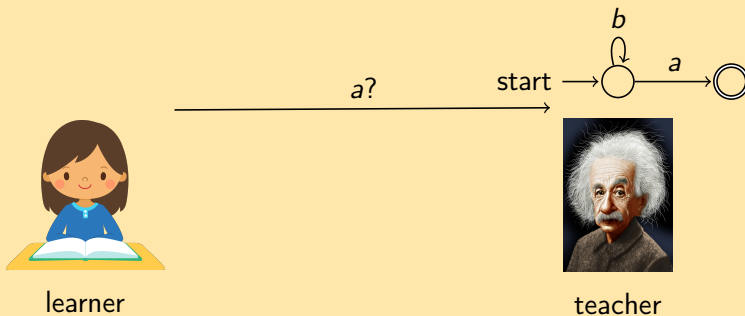
learner



teacher

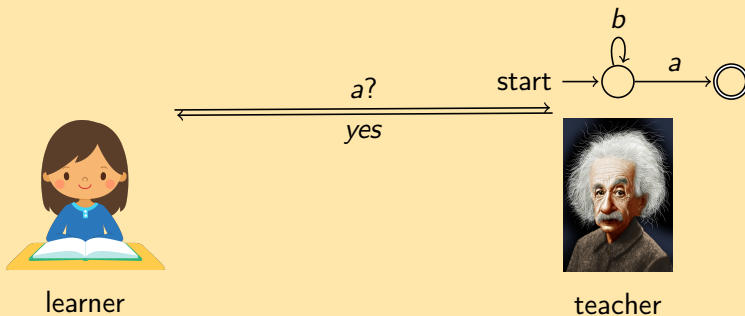
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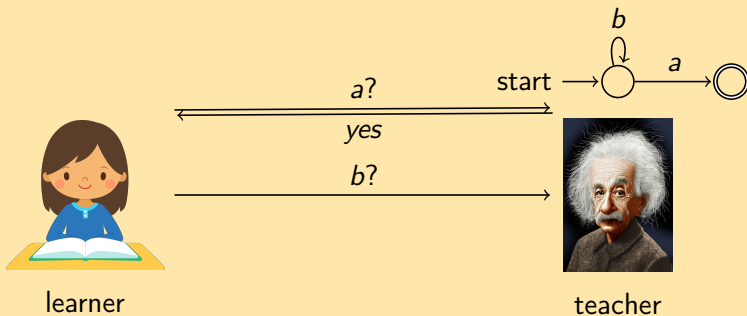
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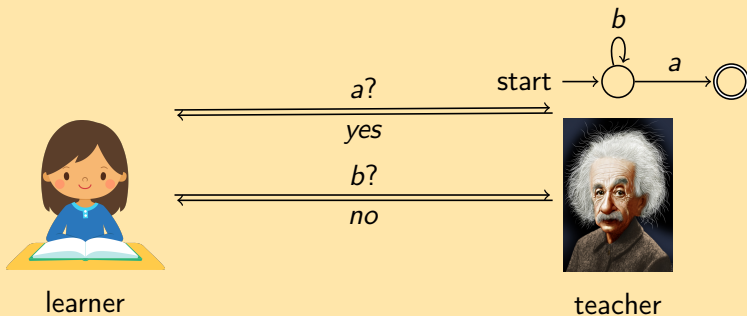
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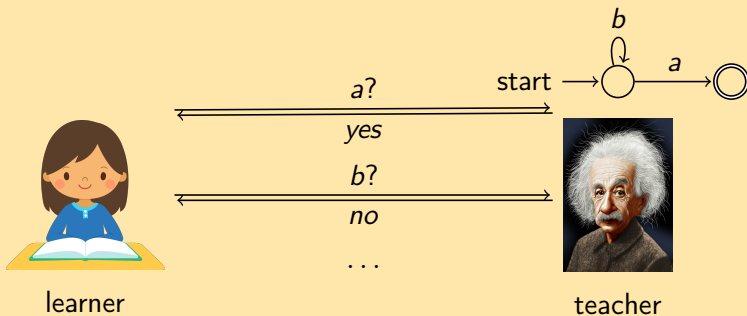
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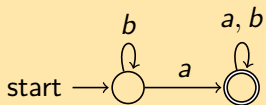
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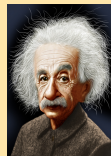
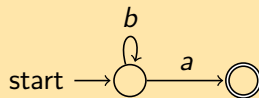


# Active learning

the minimally adequate teacher



learner

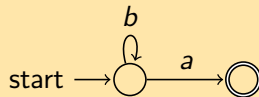
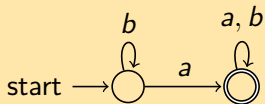


teacher



# Active learning

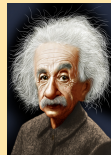
the minimally adequate teacher



learner

my model ?

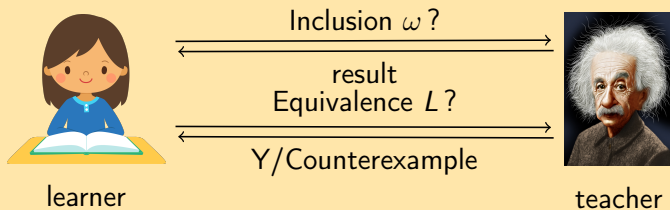
no :  $ab \notin \mathcal{L}$



teacher

# Active learning

the minimally adequate teacher



# Observation tables

|            | $\epsilon$ | $a$ | $ba$ |
|------------|------------|-----|------|
| $\epsilon$ | 1          | 0   | 0    |
| $a$        | 0          | 1   | 0    |
| $b$        | 0          | 1   | 0    |
| $aa$       | 1          | 1   | 1    |
| $ab$       | 0          | 0   | 1    |
| $ba$       | 1          | 1   | 1    |
| $bb$       | 1          | 0   | 0    |

# Observation tables

|            | $\epsilon$ | $a$ | $ba$ |   |
|------------|------------|-----|------|---|
| $R$        | $\epsilon$ | 1   | 0    | 0 |
|            | $a$        | 0   | 1    | 0 |
|            | $b$        | 0   | 1    | 0 |
| $R.\Sigma$ | $aa$       | 1   | 1    | 1 |
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closed  $\forall u \in R, \forall a \in \Sigma, \exists v \in Ru. a \sim_O v$



# Observation tables

|             |            | S          |     |      |
|-------------|------------|------------|-----|------|
|             |            | $\epsilon$ | $a$ | $ba$ |
| R           | $\epsilon$ | 1          | 0   | 0    |
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closed  $\forall u \in R, \forall a \in \Sigma, \exists v \in Ru. a \sim_0 v$

consistent  $\forall u, v \in R, (u \sim_0 v \Rightarrow \forall a \in \Sigma u.a \sim_0 v.a)$

Construct an empty observation table  $O$  ( $R = S = \emptyset$ )

**repeat**

    Make  $O$  closed and consistent (with membership queries)

    Perform an equivalence query

**if** *The teacher provided a counterexample  $u$*  **then**

        Add  $u$  to  $O$  ( $R \leftarrow R \cup u$ )

        Complete  $O$  (with membership queries)

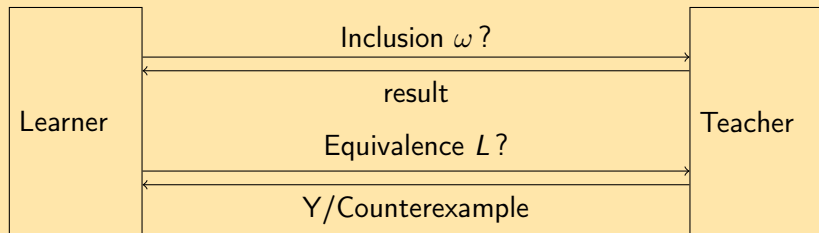
**end**

**until** *the teacher replies "yes" to an equivalence query;*

**Result:** the model constructed from  $O$

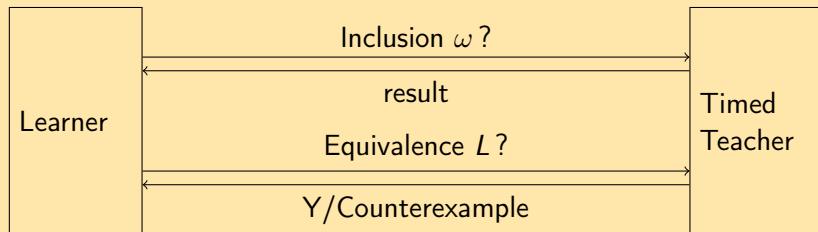
# Learning and testing

on Mealy machines with timers



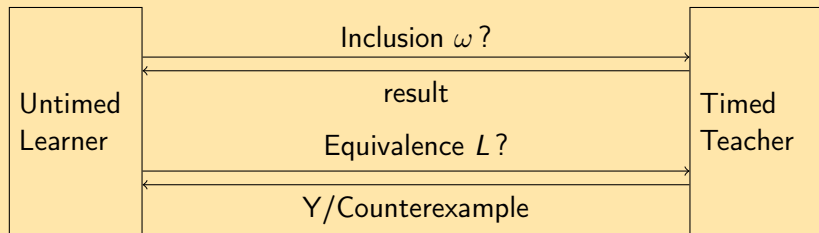
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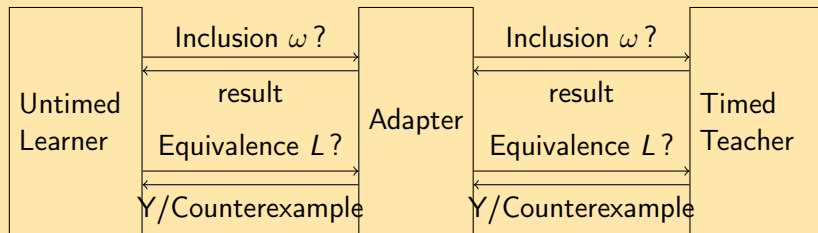
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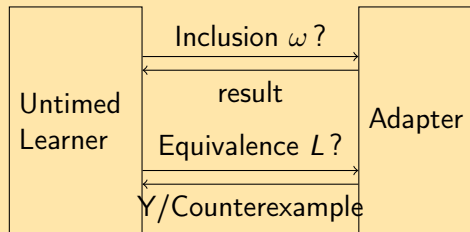
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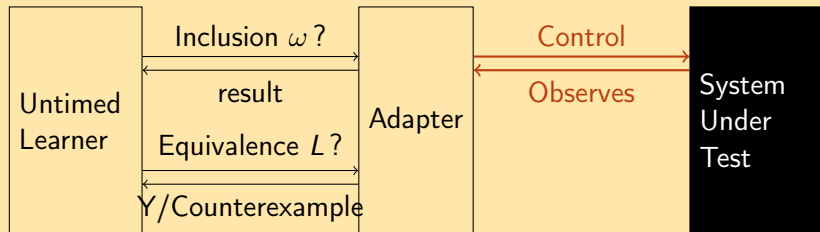
on Mealy machines with timers



System  
Under  
Test

# Learning and testing

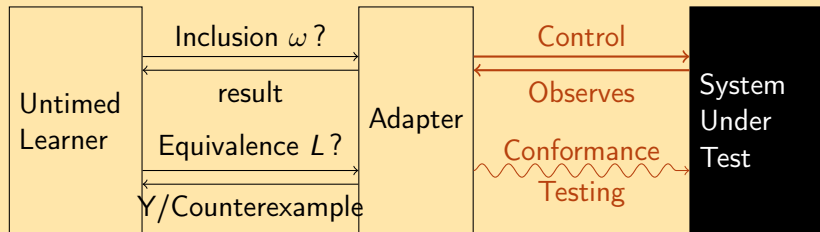
on Mealy machines with timers





# Learning and testing

on Mealy machines with timers



- ▶ Deterministic ERA can be learned by active learning;

# Event recording automata

- ▶ Deterministic ERA can be learned by active learning ;
- ▶ A very high complexity can be mitigated by targeting subclasses.

# Take Away

