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Particle Filters for Positioning with focus on Wireless Networks

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Background

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Presentation based on

- Work in competence center ISIS with
  - SAAB Aircraft: aircraft terrain aided positioning and navigation
  - SAAB Dynamics: missile tracking and torpedo positioning
  - NIRA Dynamics: Map-Aided Positioning (MAP) for cars
  - Ericsson: wireless networks
- F. Gustafsson, F. Gunnarsson, N. Bergman, U.Forssell, J. Jansson, R. Karlsson and P-J. Nordlund: *Particle filters for positioning, navigation and tracking*. IEEE Trans. On Signal Processing, 2002.
- P-J Nordlund, F. Gustafsson and F. Gunnarsson: *Particle filter for positioning in wireless networks*. EUSIPCO 2002.
- F. Gustafsson and F. Gunnarsson: *Positioning using time-difference of arrival measurements*. Submitted to ICASSP 2003.

Outline

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- Wireless positioning survey
- The particle filter for positioning
- Map-based positioning
  - Aircraft, missiles and torpedos
  - Vehicles
- Framework for wireless positioning
- TDOA based positioning

Positioning methods

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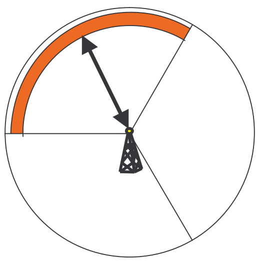
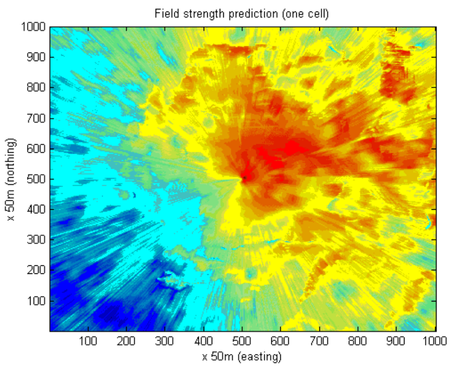
- Cell-Identifier (CI)
- Sector information or Angle of Arrival (AOA) (requires adaptive antennas)
- Timing Advance (TA) d_{TA} = ½ • 3.69µs symbol period • 3e8 m/s = 554m
- Enhancement: usage of received signal levels (RXLEV), see operator's power attenuation map below.





Trilateration: time differences in network

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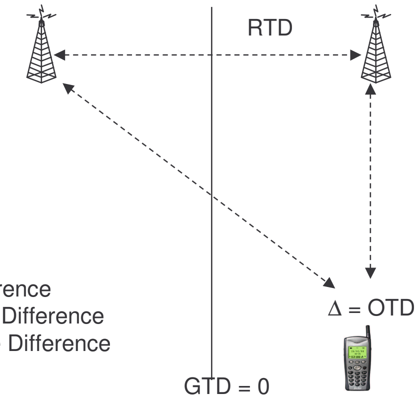

$GTD = OTD - RTD$

RTD : Real Time Difference
OTD : Observed Time Difference
GTD : Geometric Time Difference

- RTD=0 in the CDMA standard IS-95. Advanced Forward Link Trilateration (A-FLT) is standardized.
- BTSs are not synchronized in GSM (RTD ≠ 0), need Location Measurement Units. E-OTD is standardized.
- Position calculation via hyperbolic trilateration between BTS pairs

Trilateration

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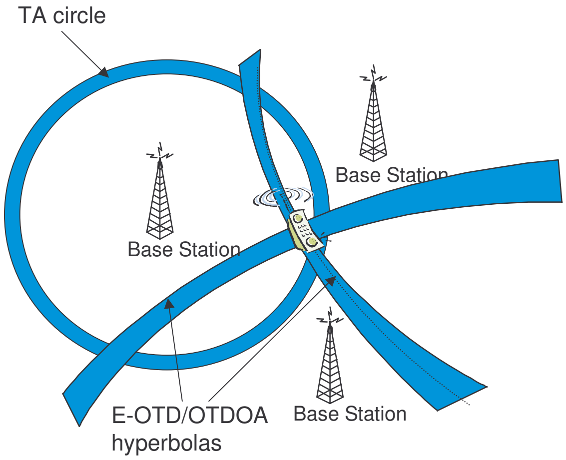
- Location Measurement Units (LMU)
  - Type A, stand-alone
  - Type B, integrated in the BTS
- LMU can be avoided by synchronizing BTSs
  - with GPS synchronization
  - by time-stamping DL emissions



Assisted GPS

- Built in GPS in Mobile Station:
  - Requires long time to first fix (60 s)
  - Energy consuming measurements
  - Computational expensive
- Assisted GPS (A-GPS)
  - Fixed stations transmit information of visible satellites and their clock correction and ephemeris (orbital corrections), spreading codes, doppler etc.
  - MS finds the information in 1s
  - Network computes position
  - Does not work indoor and in urban canyons

Positioning methods: summary

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Satellite positioning

- GPS, Global Positioning System (10-30m)
- DGPS, Differential GPS (1-10m)
- A-GPS, Assisted-GPS (1-10m GSM R99)

Cellular positioning

- COO, Cell of origin (0.2-10km GSM R99)
- TA, Timing Advance (0.5 km)
- AOA, Angle of arrival (0.1-2km)
- TOA, Time of arrival (100-200m GSM R99)
- E-OTD, Enhanced Observed Time Difference (50-200m GSM R99)

In-door positioning

- WLAN, Bluetooth, GPS-pseudolites

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Expected Revenues

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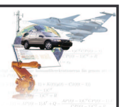
Localised Services	Strategies Group	Ovum
Billing	~$Bio 15.9	
Information	~$Bio 13.5	~$Bio 1.9
Roadside assistance	~$Bio 1.7	
Traffic & navigation	~$Bio 1.6	
M-Commerce		~$Bio 1.1
Advertising		~$Bio 2.1
2005 EU operator revenues	**~$Bio 32.7**	**~$Bio 5**
Y00-Y05 cumulated revenues	**~$Bio 81.9**	**~$Bio 11**

Certainly quite uncertain predictions!

Models



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- Model when velocity is measured:

$$x_t = (X_t, X_t, \Psi_t)^T$$
$$X_{t+1} = X_t + Tv_t \cos(\Psi_t) + w_{X,t}$$
$$Y_{t+1} = Y_t + Tv_t \sin(\Psi_t) + w_{Y,t}$$
$$\Psi_{t+1} = \Psi_t + T\dot{\Psi}_t$$
$$y_t = h(X_t, Y_t) + e_t$$

- Model when velocity is <u>not</u> measured (then estimate it!):

$$x_t = (X_t, \dot{X}_t, Y_t, \dot{Y}_t)$$
$$x_{t+1} = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} x_t + \begin{pmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{pmatrix} w_t$$
$$y_t = h(X_t, Y_t) + e_t$$

Bayesian filtering

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- State space model:

$$x_{t+1} = f(x_t) + w_t$$
$$y_t = h(x_t) + e_t$$

- Bayesian time update and measurement update

$$p(x_{t+1} | Y_t) = \int p(x_{t+1} | x_t) p(x_t | Y_t) dx_t$$
$$= \int p_w(x_{t+1} - f(x_t)) p(x_t | Y_t) dx_t$$

$$p(x_t | Y_t) = \frac{p(y_t | x_t) p(x_t | Y_{t-1})}{p(y_t | Y_{t-1})}$$
$$= C p_e(y_t - h(x_t)) p(x_t | Y_{t-1})$$

PF Algorithm

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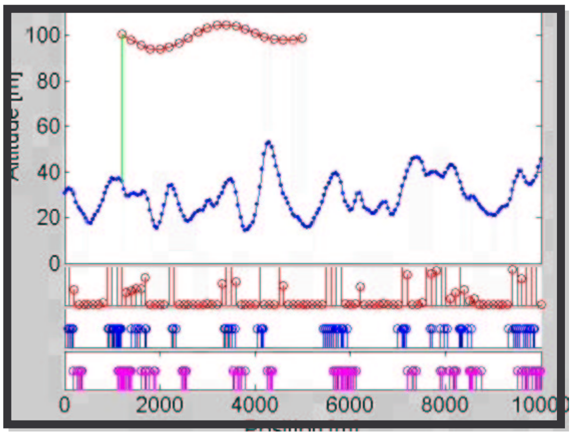
Example: Terrain navigation in 1D



Generic Particle Filter

1. Generate random states $x_0^{(i)} \in p(x_0)$
2. Compute likelihood

$$\omega_t^{(i)} = p_e(y_t - h(x_t^{(i)}))$$

3. Resampling: $x_t^{(i)} \propto \omega_t^{(i)}, \omega_t^{(i)} = \frac{1}{N}$
4. Prediction: $x_{t+1}^{(i)} = f(x_t^{(i)}) + w_t^{(i)}, w_t^{(i)} \in p_w$

Note:

1. **Cramer-Rao: position error > sqrt (altitude error * velocity error / terrain variation)**
2. **The particle filter normally attains this bound!**

GIS database

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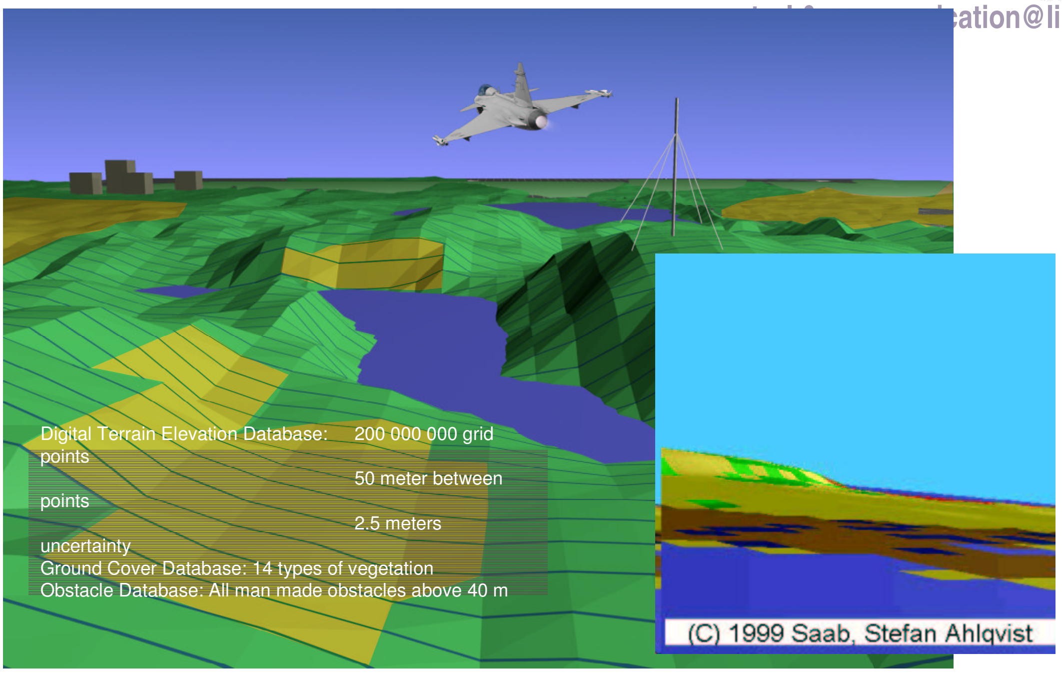

2D Example

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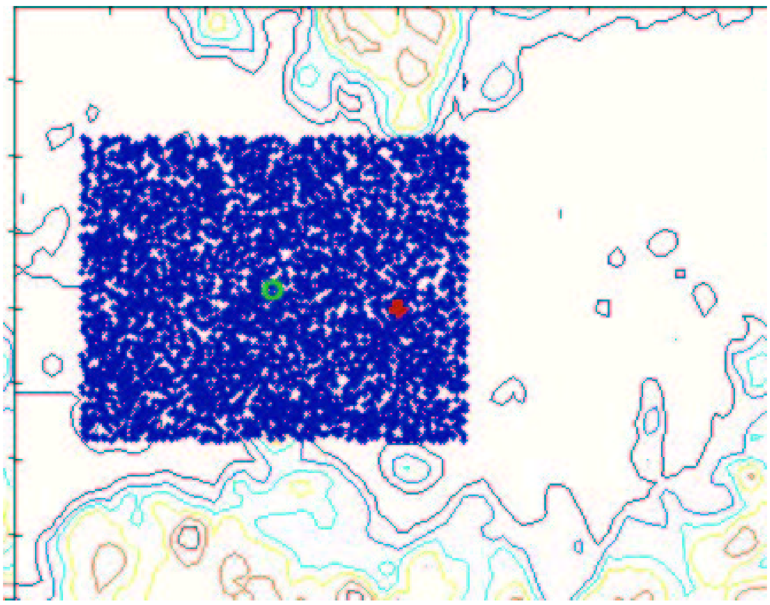
Animation of terrain navigation in 2D using real GIS



Car positioning

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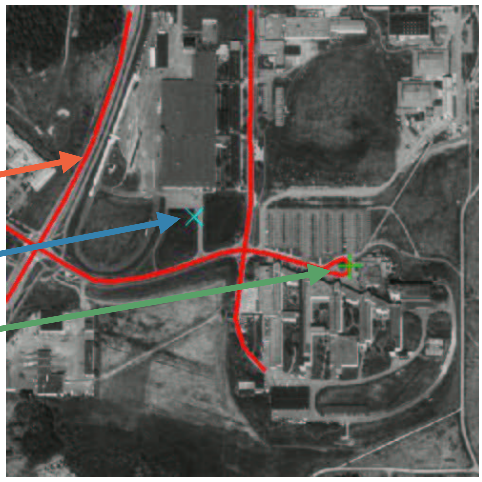
- Initization using manual marking or GSM positioning

Particles

Position estimate

True position



Car positioning

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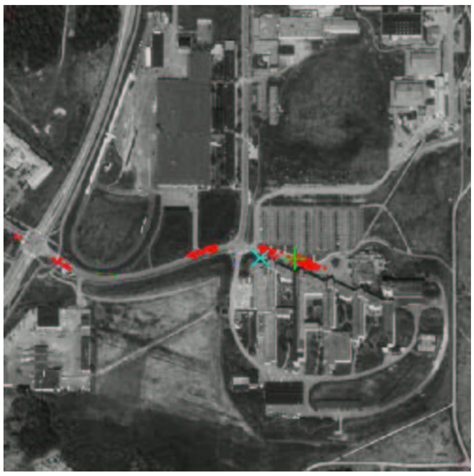
- Initization using manual marking or GSM positioning
- After slight bend, four particle clusters left



Car positioning

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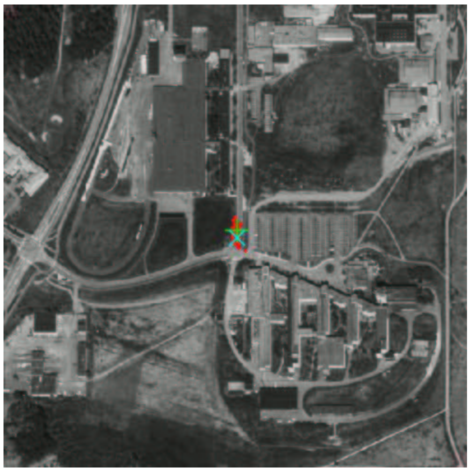
- Initizalization using manual marking or GSM positioning
- After slight bend, four particle clusters left
- Convergence after turn



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Car positioning

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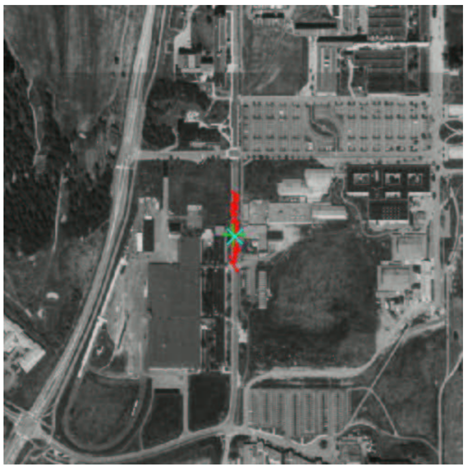
- Initizalization using manual marking or GSM positioning
- After slight bend, four particle clusters left
- Convergence after turn
- Spread along the road



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Car positioning

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- Particle filter using street map and v(t),$\dot{\Psi}(t)$ from car's ABS sensors.
- Green - true position
- Blue – estimate
- Red - particles

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Car positioning

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- Particle filter using street map and v(t),$\dot{\Psi}(t)$ from car's ABS sensors and <u>GSM cell ID and sector</u> for initialization
- Purple: GPS
- Blue: particles
- Light blue: estimate
- Photo background

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Car positioning

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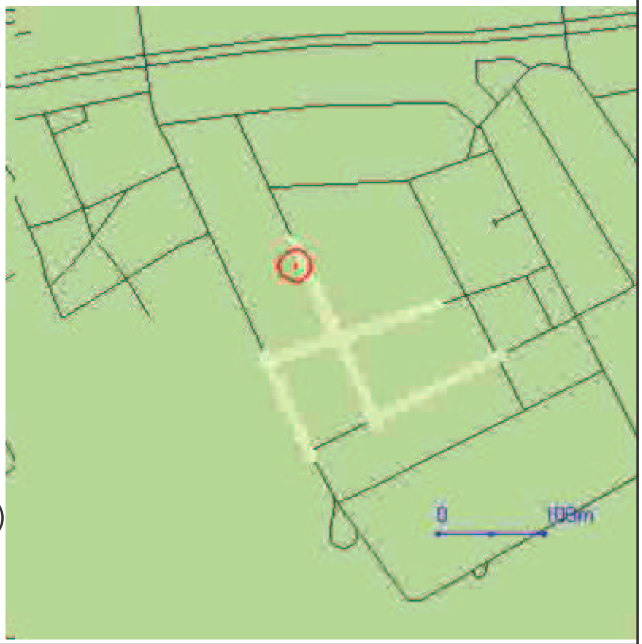
- Red – GPS
- Light green: particles
- Blue: estimate (after convergence)
- Real-time implementation on Compac iPAQ
- Works without or with GPS
- Map database background
- Complete navigator with voice guidance!
- Integer implementation of the particle filter (ISIS project)
- PF in simulation mode off-road
- # particles up to 15000 (without GPS) or as small as 50 (with GPS)
- On-going R&D work at NIRA Dynamics AB and ISIS



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Network measurements

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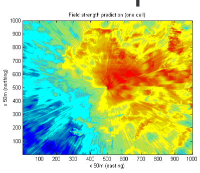
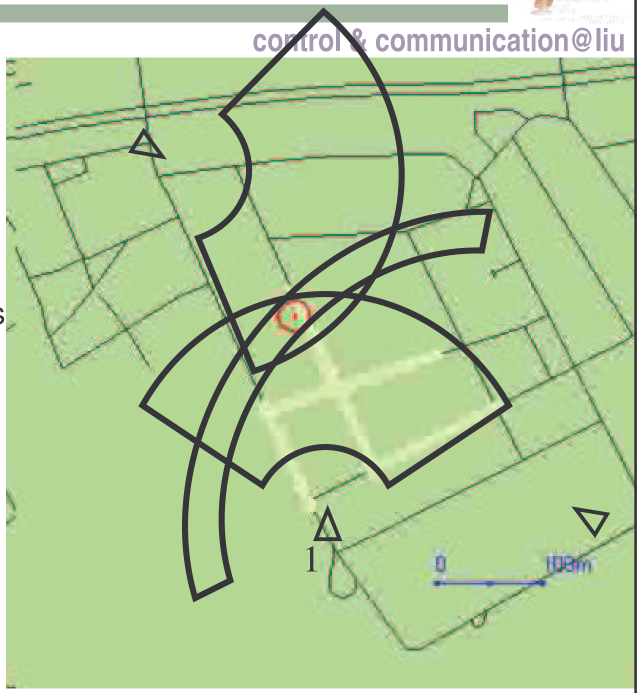
1. Cell ID and Received signal strength:

- Sector information (usually 60 degrees)
- One or more sectors:
  - Connected antenna (yellow pages service)
  - Power measurements from 5 (GSM) or 6 (WCDMA) antennas
- Power attenuation: Hata's formula or map-based





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Network measurements

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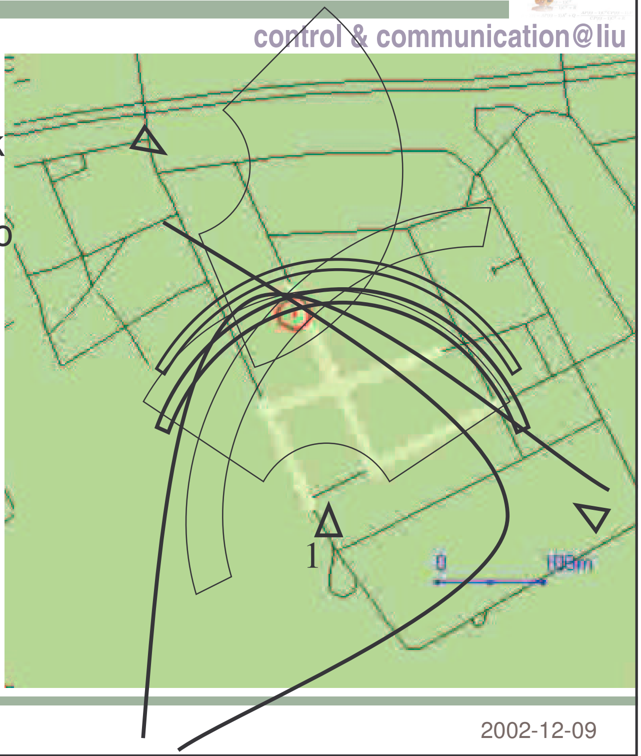
2. Time of arrival

- Time of arrival (TOA) in uplink gives time delay in uplink transmission. Uncertain due to multipath.
- Time difference of arrival (TDOA) measures downlink time differences.
- Enhanced observed time difference (E-OTD) as TDOA but for unsynchronized base stations.



Information (sensor) fusion

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Positioning of mobiles/cars using two out of three of the following information sources:

1. Road map (vectorized) penalizes positions off-road
2. Velocity vector: ABS sensor measuring wheel speed in cars gives v(t) and $\dot{\Psi}(t)$
3. Wireless network parameters and antenna positions:

– TOA (time of arrival)

– TDOA (time difference of arrival)

– E-OTD (enhanced time difference of arrival)

– AOA (angle of arrival)

These can all be described by measurement equation $y_t = h(X_t, Y_t) + e_t$ with a certain error distribution

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Reducing the complexity

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- Complexity increases in state dimension, but **not exponentially** as for point mass filter and other numerical approximations.
- Split state vector in position and other states.

$$\begin{pmatrix} x^{\mathrm{pf}}_{t+1}\\ x^{\mathrm{kf}}_{t+1}\end{pmatrix} = \begin{pmatrix} I & A^{\mathrm{pf}}\\ 0 & A^{\mathrm{kf}}\end{pmatrix}\begin{pmatrix} x^{\mathrm{pf}}_t\\ x^{\mathrm{kf}}_t\end{pmatrix} + \begin{pmatrix} B^{\mathrm{pf}}_u\\ B^{\mathrm{kf}}_u\end{pmatrix} u_t + \begin{pmatrix} B^{\mathrm{pf}}_f\\ B^{\mathrm{kf}}_f\end{pmatrix} f_t$$

$$y_t = h(x^{\mathrm{pf}}_t) + e_t,$$

- Marginalization implies that KF can be used for linear part.

Marginalization

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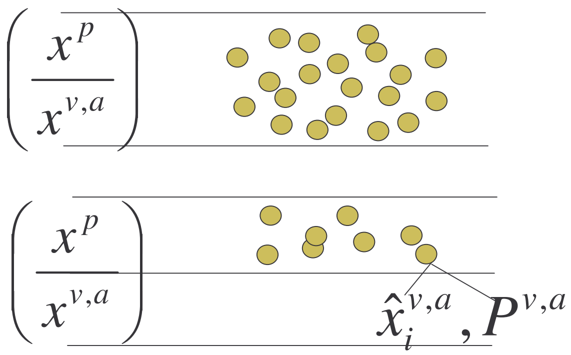
- PF on all states
- PF on position states, KF on the other ones
- Same Riccati equation for all KF's



1D Example

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- PF on the model:
  requires > 20000 particles

$$x_{t+1} = \begin{pmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{pmatrix} x_t + \begin{pmatrix} \frac{T^3}{6} \\ \frac{T^2}{2} \\ T \end{pmatrix} w_t$$

$$y_t = h(x_t^1) + e_t.$$

- PF on the model:
  requires < 1000 particles

$$x_{t+1}^1 = x_t^1 + \begin{pmatrix} T & \frac{T^2}{2} \end{pmatrix} x_t^2 + \left(\frac{T^3}{6}\right) w_t$$

$$x_{t+1}^2 = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} x_t^2 + \begin{pmatrix} \frac{T^2}{2} \\ T \end{pmatrix} w_t.$$

TDOA estimation problems

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- The mathematical problem setup
- Static positioning, using optimization
- Dynamic positioning using dynamic motion model and filtering

Mathematical problem

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- Aligned antennas at (-D/2,0) and (D/2,0) giving GTD=d gives non-linear equation for position

$$d = \sqrt{y^2 + (x+D)^2} - \sqrt{y^2 + (x-D)^2}$$

- Corresponding equation for hyperbola:

$$\frac{x^2}{d^2/4} - \frac{y^2}{D^2/4 - d^2/4} = 1$$

- Transition to global coordinate system gives for BTS pair (i,j):

$$d_{i,j} = h(X, Y, X_i, Y_i, X_j, Y_j) = h(P, P_i, P_j)$$

- Several GTD's provide non-linear equation system
- Measured time has Rice/Rayleigh noise □ Cov(**d**) non-diagonal

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TDOA problem setup

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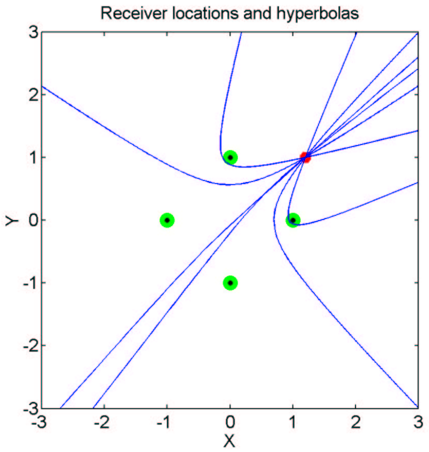
- n BTS provides $\binom{n}{2}$ hyperbolas
- Each pair of hyperbolas can have 0,1,2,3 or 4 intersections!
- Approaches:

1. Solve these non-linear equations pair-wise
2. Use the particle filter for static optimization
3. Solve a non-linear least squares problem numerically

$$\hat{P} = \arg\min_P \mathbf{h}^T(P, \{P_i\}_{i=1}^n) R^{-1} \mathbf{h}(P, \{P_i\}_{i=1}^n)$$

Only 2 can be generalized to dynamic models



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TDOA static positioning

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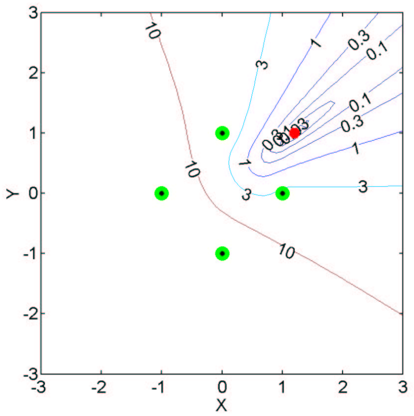
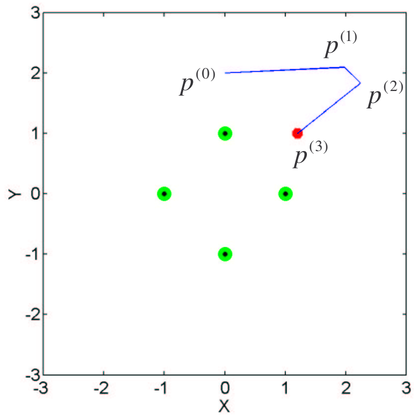
- The least squares error is for this configuration a unimodal function. Steepest descent algorithms work for all initializations.





TDOA static positioning

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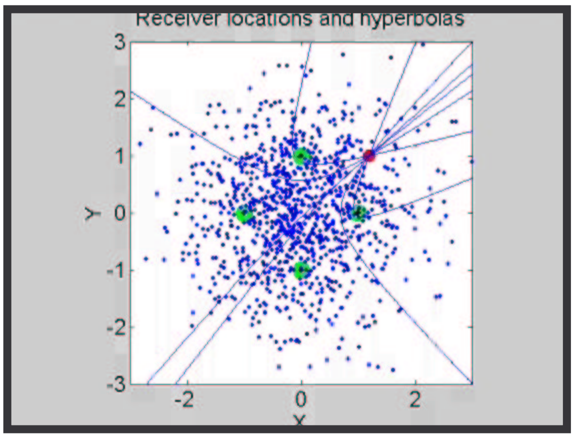
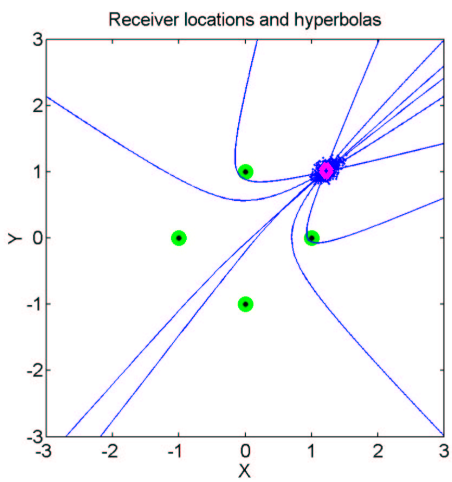
- Particle filter with decaying roughening noise





Simulations

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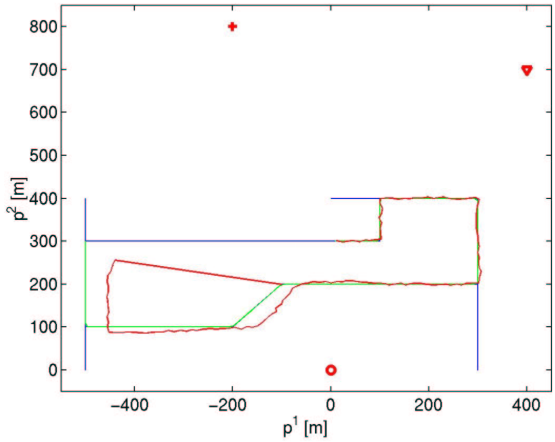
Simulation setup

- Artificial street map
- Known velocity
- Power measurement (std 6 dB) and TOA distance measurement (std 3dB) with random walk biases
- Bias are either estimated or not
- The no bias case investigated for comparison
- 2000 particles, 50 Monte Carlo runs



Simulations

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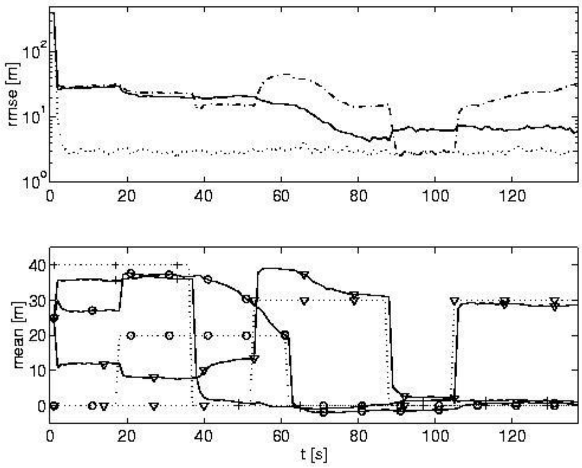
Results:

- Upper plot shows RMSE(t) when bias estimated (solid) and not estimated (dashed). Dotted line is for simulations without bias.
- Lower plot shows bias estimate.



Conclusions

- Positioning of cellular phones using network measurements
- Focus on dynamic estimation and automotive application
- Flexible framework configurable with an asynchronous mixture of information sources:
    1. Map: street, altitude, attenuation
    2. Angle and range to fix-point: AOA, TOA, TDOA, GPS
    3. Velocity and turn rate: $v(t), \dot{\Psi}(t)$
- Particle filter suitable because of its ability to include:
    1. Non-linear constraints (map)
    2. Non-Gaussian noise (Rice and Rayleigh fading, operator's power attenuation map, etc.)