Ray Tracing

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Introduction to Computer Graphics

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Context

- Representation and visualization of things which do not exist
  - Create 3D virtual worlds
    - Choose the model of the objects we want to represent
  - Visualize them
    - Choose a representation model
  - Animate these worlds

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Interest in 3D virtual worlds

- Computer aided design
- Manufacturing
- Movies
- Entertainement: video games
- Data Visualization
- Virtual Reality
- Working without risk
- Simulation

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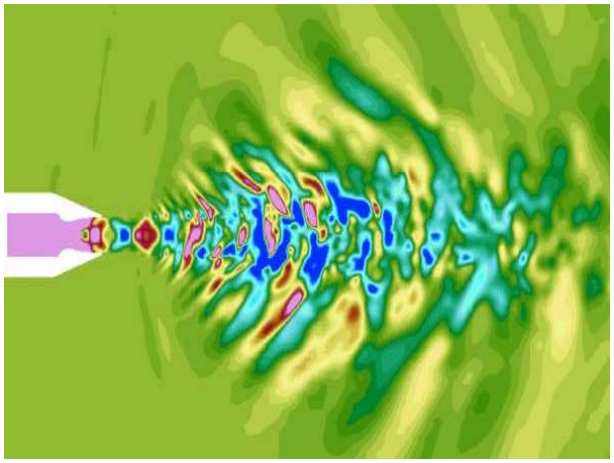
Application



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Application

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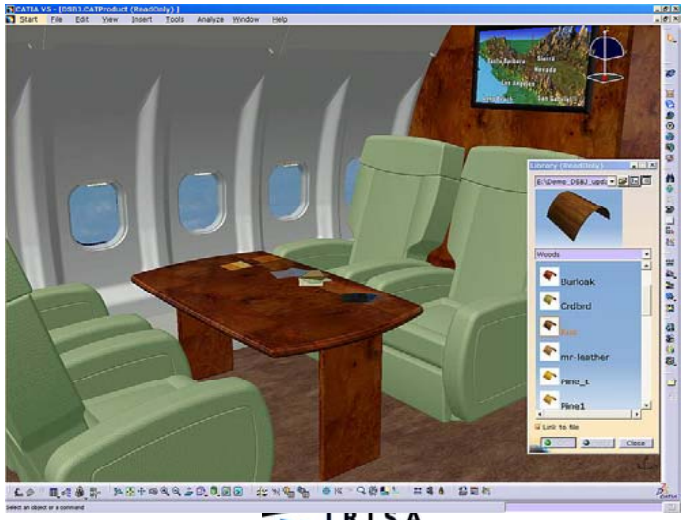
Application



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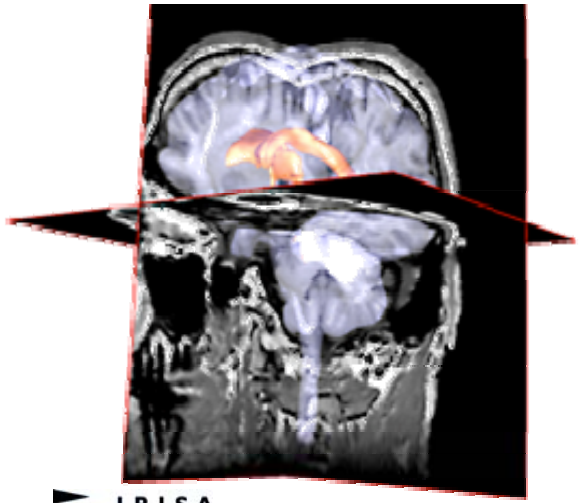

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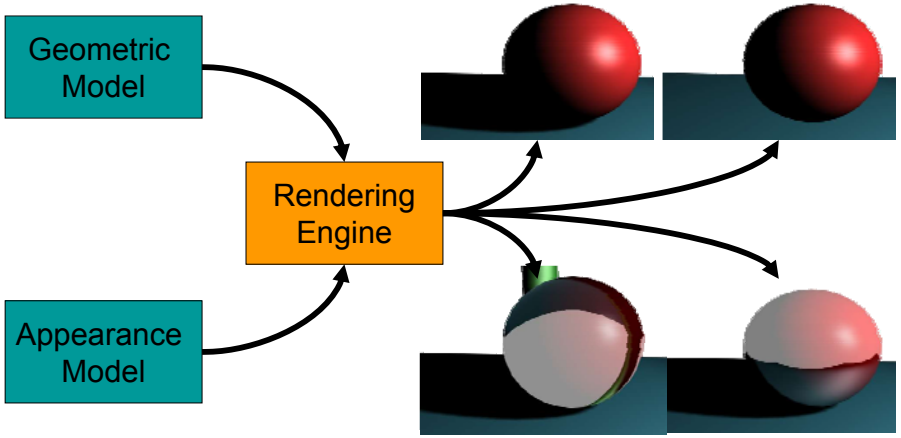
Rendering Engine



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Rendering Engine

- Real-time
  - Z-buffer
    - 10 images per second >> 0,1s
    - 25 images per second >> 0,04 s
    - 60 images per second >> 0,016 s
    - 120 images per second >> 0,008 s
- Non real-time
  - Ray Tracing, Path Tracing, Photon Mapping
  - Radiosity

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Illumination Model

- Expresses the light intensity at a point due to:
  - The light sources giving rise to :
    - diffuse and specular reflections
    - shadows
  - The reflectance and the transmittance of the object
- $I_r = I_d + I_s$
  - I_d : diffuse intensity
  - I_s : specular intensity

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Diffuse Reflection

- Lambert Reflection (Diffuse):
  - Part of the incident light penetrates the object then arises from the object with the same intensity in all the directions
  - $I_d = K_d \cdot I_{source} \cdot \cos(N, L) / d^2$
    - K_d : diffuse color of the object
    - I_{source} : intensity of the light source
    - N : normale at point P
    - L : light direction
    - d : distance between the light source and point P
  - Related to the microscopic roughness of the object :
    - The more K_d is high, the more important is diffusion

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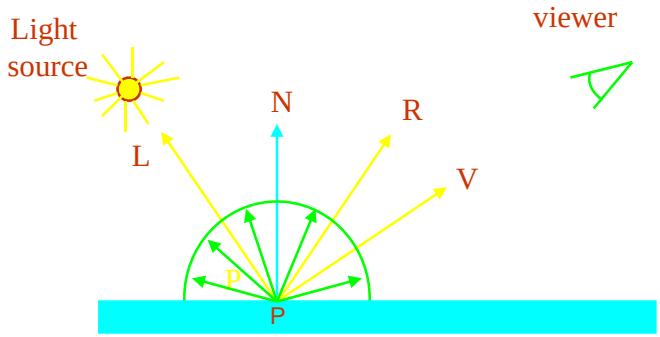
Diffuse Reflection

- $I_d = K_d \cdot I_{source} \cdot \cos(N, L) / d^2$
- d = distance of the light source to the lit point P



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Specular Reflection

- Phong Model:
  - reflection by the object's surface of the part of the incident light that did not penetrates the object; it depends on the view direction V
  - $I_s = K_s \cdot I_{source} \cdot \cos^n(R, V) / d^2$
    - K_s : specular color of the object
    - I_{source} : intensity of the light source
    - R : direction of ideal specular reflection
    - V : view direction
    - n : shininess or roughness
      - n high: shiny, tight reflection cone
      - n small: mat object, wide reflection cone
      - For glass, n = 200
      - Gives the size of highlight

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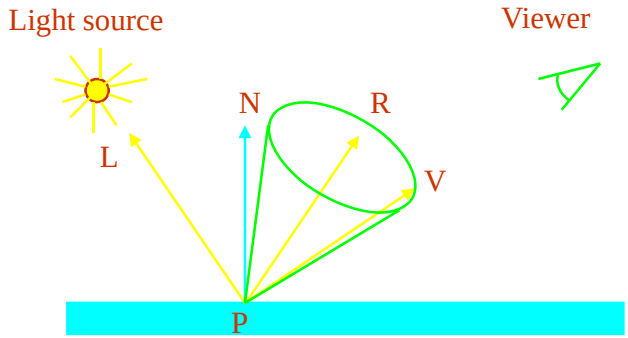
Specular Reflection

- $I_s = K_s \cdot I_{source} \cdot \cos^n(R, V) / d^2$



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The Phong Illumination Model

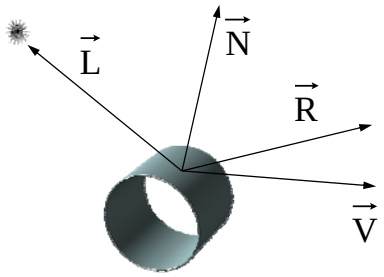
$$I_{local} = \sum_{i=0}^{nbLum} I_i \times \frac{vis(i)}{d_i^{\,2}} \times \left(k_d \left(\vec{N} \cdot \vec{L_i} \right) + k_s \left(\vec{R_i} \cdot \vec{V} \right)^n \right)$$

K_s : specular color (R,G,B)

K_d : diffuse color (R,G,B)

L_i : lighting direction of source i

I_i : intensity of source i (R,G,B)



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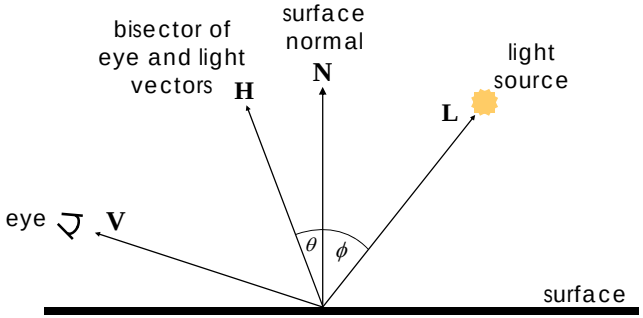
The Phong Reflection Model



$$R = I_l \left(k_d \cos(\phi) + k_s \cos^n(\theta) \right) + k_d Ia$$

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Some Remarks …

- If multiple sources, sum their contributions
- Several directions have to be known
  - The normal to the objects, the light source directions and the view direction:
    - the directions L, N et R are coplanar
    - The angle (L, N) is equal to (N, R)
- K_s and K_d are 3-component vectors:
  - Red, Green, Blue
- $I_d + I_s = (k_d \,.\, \cos(N, L) + k_s \,.\, \cos^n(R, V)) \,.\, I_{source} / d^2$
  - The term between () = BRDF

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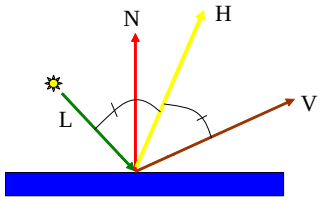
Some Remarks …

- $I_s = ks \,.\, \langle N,H \rangle^n \,.\, I_{source}$
- If the surface is perfectly specular, n is very large
- $\langle N,H \rangle^n$ is not negligible only for (N,H) = 0
- Thus Ir = ks . I_{source}
- (N,H) = 0 means that the incident and reflection angles are equal
- Only 1 reflected ray: because we assume the surface perfectly specular



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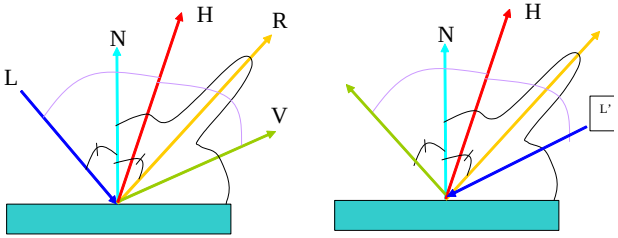
Some Remarks …



- Suppose (L',N) = (V,N) and (V',N) = (L,N)
- Then : (N,H) = (N,H')
- Ir = ks . $\langle N,H \rangle^n$. Is
- Ir' = ks . $\langle N',H' \rangle^n$. Is'
- Thus : ks . $\langle N,H \rangle^n$ = ks . $\langle N,H' \rangle^n$
- This is the reciprocity of the reflection model

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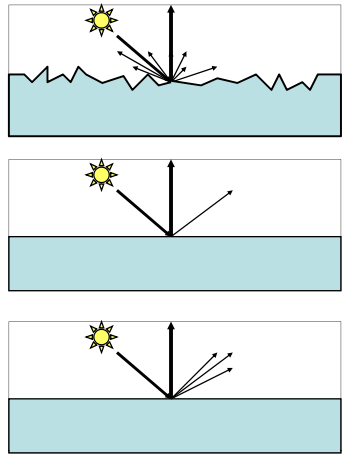
Reflections

- We normally deal with a perfectly diffuse surface.
- With ray-tracing, we can easily handle perfect reflections.
- Phong allows glossy reflections of the light source.



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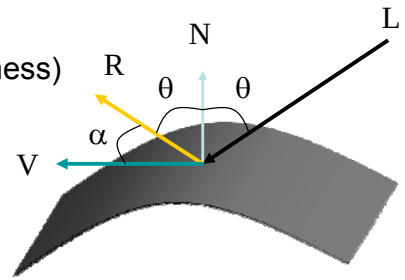
To sum up

- Division of the reflectance into 2 components
  - Diffuse
    - one 3D vector k_d
  - Specular
    - one 3D vector k_s
    - one coefficient n (shininess)



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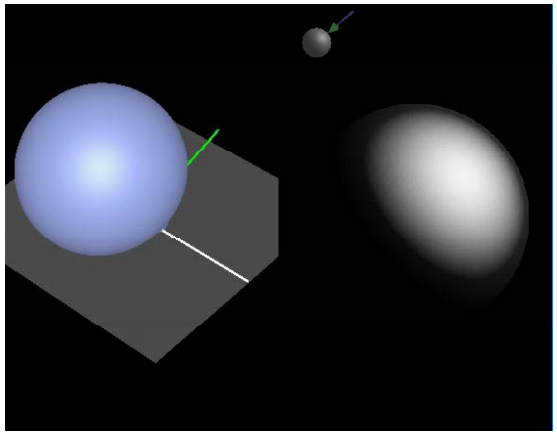
Example

•Parameters :
- K_d =0.25
- K_s =0.75
- n=1.0



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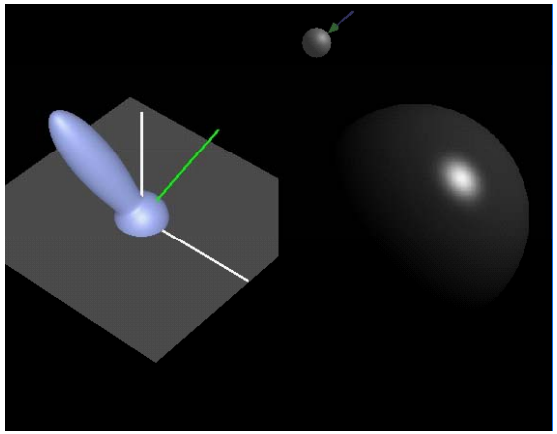
Approximation de la réflectance

•Paramètres :
- K_d =0.25
- K_s =0.75
- n=50.0



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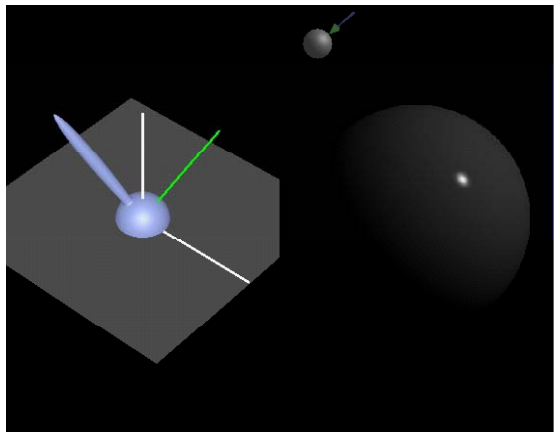
Approximation de la réflectance

•Paramètres :
- K_d =0.25
- K_s =0.75
- n=200.0



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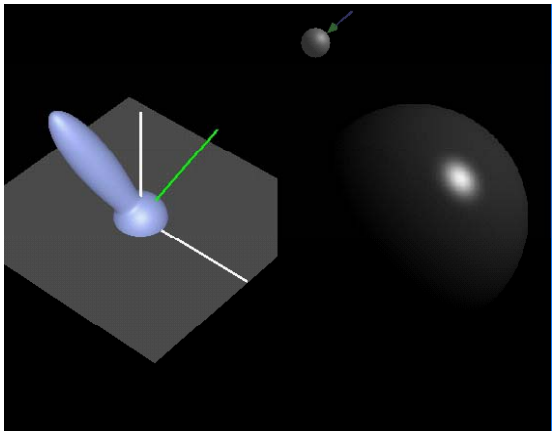
Approximation de la réflectance

- Paramètres :
  - K_d =0.25
  - K_s =0.75
  - n=50.0



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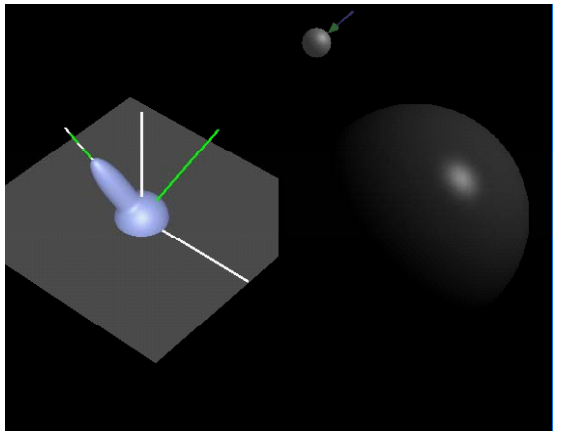
Approximation de la réflectance

- Paramètres :
  - K_d =0.25
  - K_s =0.25
  - n=50.0



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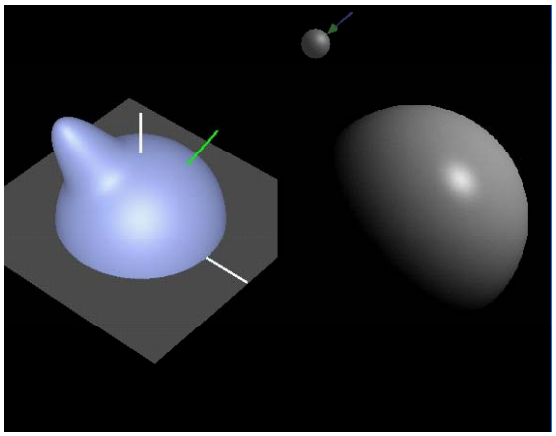
Approximation de la réflectance

- Paramètres :
  - K_d =0.75
  - K_s =0.25
  - n=50.0



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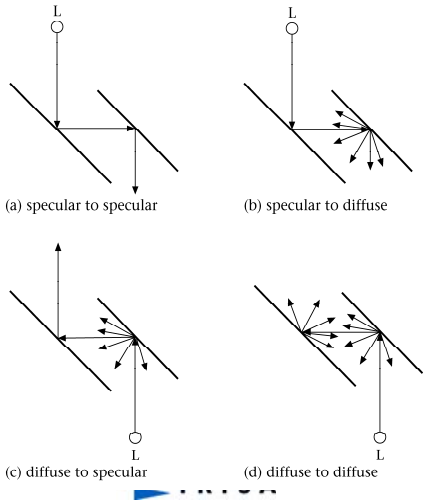
Recap: Different Light Transports



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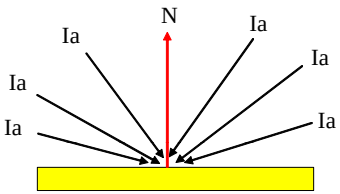
Ambient Term

-The indirect diffuse component due to multiple reflections is supposed to be the result of the diffuse reflection of an ambient term Ia



- $I_{id} = k_d \cdot I_a$

- Ia is the same for all the surfaces

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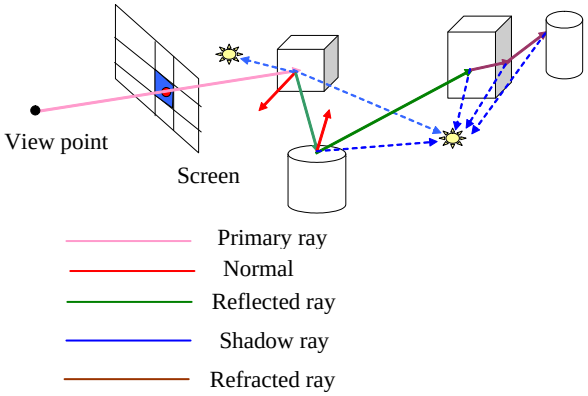
Principle



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Principle

- Trace a primary ray passing through a pixel
- P : intersection point
- Compute the contribution of the sources to P by tracing shadow rays toward the light sources.
- If a shadow ray intersects an opaque object between P and the light source then P is shadowed
- Compute the contribution to P of other points within the scene by tracing secondary rays: reflected and refracted
- A reflected ray is traced only if the material is specular
- A refracted ray is traced only if the material is transparent
- A secondary ray intersects the scene at a point P'
- Again compute the contribution of the sources to P' by tracing shadow rays toward the light sources.
- Repeat the process
- Each ray brings its contribution to the luminance of a point

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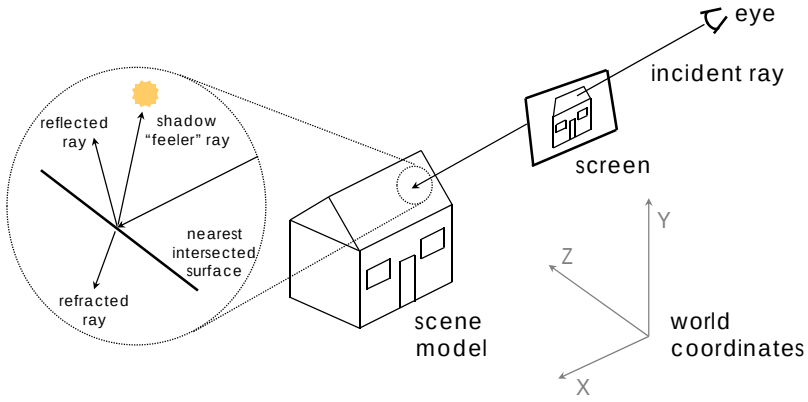
Principle



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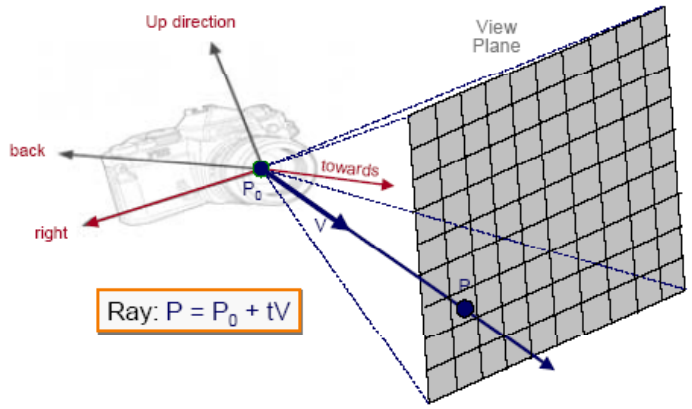
Principle



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Principle

2D Example

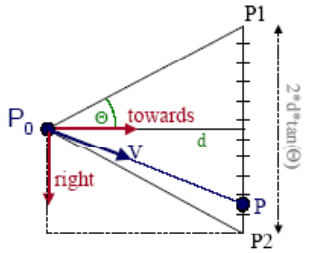
right = towards x up

Θ = frustum half-angle
d = distance to view plane

P1 = P0 + d*towards – d*tan(Θ)*right
P2 = P0 + d*towards + d*tan(Θ)*right

P = P1 + (i/width + 0.5) * (P2 - P1)
  = P1 + (i/width + 0.5) * 2*d*tan (Θ)*right

V = (P - P0) / ||P - P0 ||



Ray: P = P0 + tV

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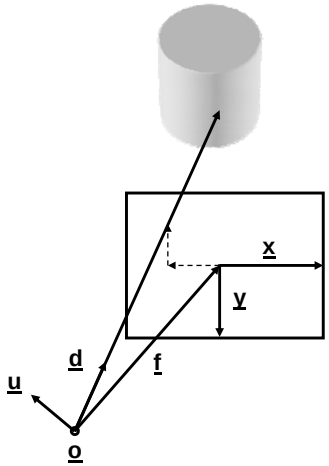
Ray Generation

- Pinhole camera

```
for (x= 0; x < xres; x++)
  for (y= 0; y < yres; y++)
  {
    d= f + 2(x/xres - 0.5)·x
         + 2(y/yres - 0.5)·y;
    d= d/|d|; // Normalize
    r.d = d; r.o = o ;
    color= ray_cast(r,scene,depth);
    write_pixel(x,y,color);
  }
```



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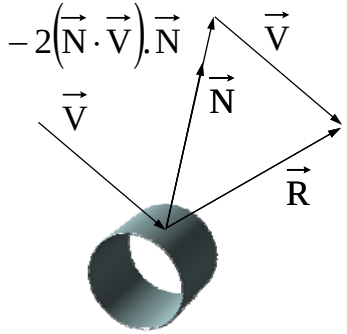
REFLECTION



$$\vec{R} = -2(\vec{N}\cdot\vec{V}).\vec{N} + \vec{V}$$

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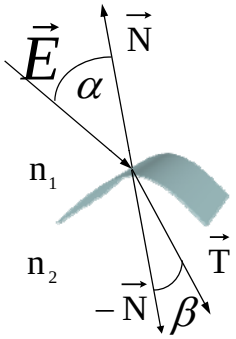
REFRACTION



$$n_1 \sin(\alpha) = n_2 \sin(\beta)$$

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Refraction: Using Snell's Law

$$\frac{\sin\alpha}{\sin\beta} = \frac{\eta_2}{\eta_1} = \eta_{21}$$

- Using this law it is possible to show that:

$$T = -\eta_{12}E + N\left(\eta_{12}\cdot\cos\alpha - \sqrt{1+\eta_{12}^2\cdot\left(\cos^2\alpha - 1\right)}\right)$$

- Note that if the root is negative then total internal reflection has occurred and you just reflect the vector as normal

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Ray-Tracing: Pseudocode

- For each ray **r** from eye to pixel, color the pixel with the value returned by ray_cast(**r** , scene,depth):

```
ray_cast(r, scene,depth)
{
   If(depth >Max_Depth) {color ← black}
   else {
      If (intersection(r,scene)) {
      p ← point_of_intersection(r, scene);
      u← reflect(r, p);
      v← refract(r, p);
      color ← phong_direct(p, r) +
           k_s × ray_cast(u, scene, depth+1) +
           kt× ray_cast(v, scene , depth+1);
       } else color ← background_color ;
    }
return(color);
}
```

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Pseudocode Explained

- **p** ← point_of_intersection(**r**, scene);
  - Compute **p**, the point of intersection of ray **r** with the scene
- **u**← reflect(**r**, **p**); **v**← refract(**r**, **p**);
  - Compute the reflected ray **u** and the refracted ray **v** using Snell's Laws

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Pseudocode Explained

- phong(**p**, **r**)
  - Evaluate the Phong reflection model for the ray **r** at point **p** on surface s, taking shadowing into account
- $k_s \times$ ray_cast(**u,scene,depth**)
  - Multiply the contribution from the reflected ray **u** by the specular color k_s for surface s containing **p.** **Only (specular-to-specular)* light transport is handled. Ideal specular (mirror) reflection**
- $k_t \times$ ray_cast(**v,scene,depth**)
  - Multiply the contribution from the refracted ray **v** by the specular-refraction coefficient k_t for surface s. **Only (specular-refraction)* light transport is handled**

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About Those Calls to ray_cast()...

- The function ray_cast() calls itself *recursively*
- There is a potential for infinite recursion
  - Consider a "hall of mirrors"
- Solution: limit the depth of recursion
  - A typical limit is five calls deep
  - Note that the deeper the recursion, the less the ray's contribution to the image, so limiting the depth of recursion does not affect the final image much

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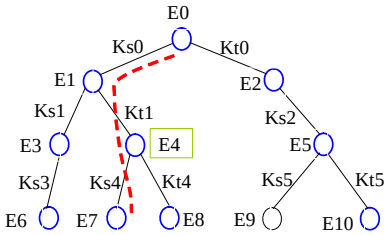
About Those Calls to ray_cast()...

- Another solution
  - Ei: direct lighting at point Pi
  - Ks: vector (R,G,B)
  - Kt: scalar ranging between 0 and 1
  - Contribution of the red path



- I : Intensity due to this ray path :

I = Ks0 . (Kt1 (Ks4 . E7 + E4) + E1)
= Ks0 . Kt1 . Ks4 . E7 + Ks0 . .Kt1 . E4 + Ks0 . E1

- Stop tracing rays when the cumulative product Ks.Kt… is below a certain threshold

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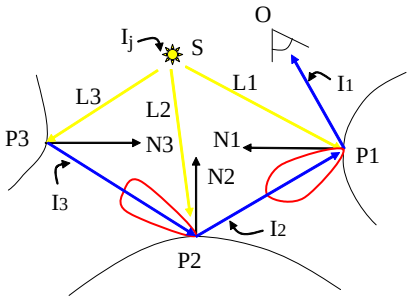
Example

H1 : bisecting line of angle S P3 P2
H2 : bisecting line of angle S P2 P1
H1 : bisecting line of angle S P1 O
Idai : intensity due to direct lighting and the ambient term for point Pi

Idai = kdi . Ia
+ kdi . Is . cos(Li,Ni)
+ ksi . Is . cos(Ni,Hi)^n

I3 = Ida3
I2 = Ida2 + ks2 . I3
I1 = Ida1 + ks1 . I2



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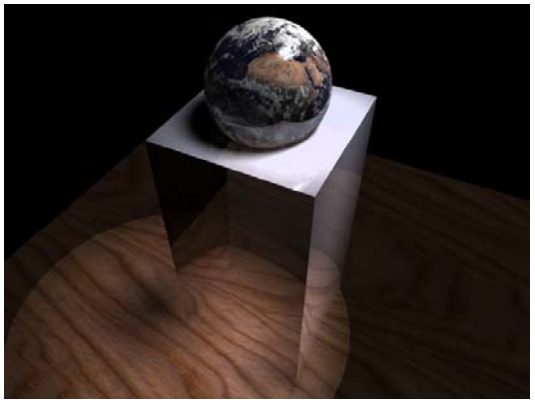
Reflections

- If only one reflected ray is considered, then ray-tracing will only handle perfect mirrors.



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Reflections

- Glossy reflections (multiple reflected rays) blur the reflection.

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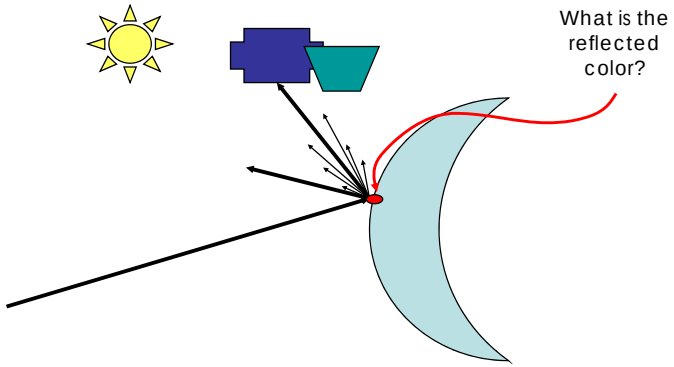
Reflections

- Mathematically, what does this mean?



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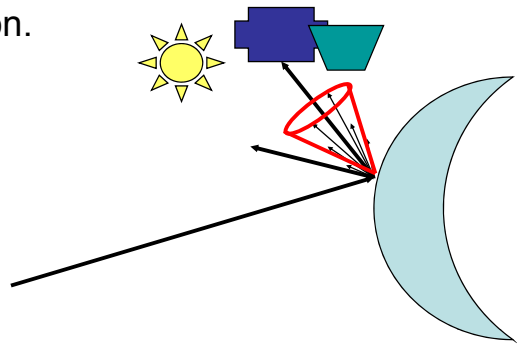
Glossy Reflections

- We need to integrate the color over the reflected cone.
- Weighted by the reflection coefficient in that direction.



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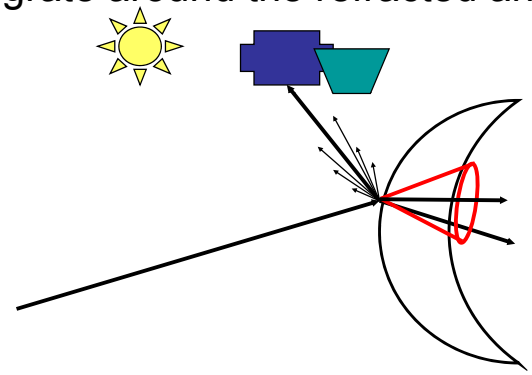
Translucency

- Likewise, for blurred refractions, we need to integrate around the refracted angle.



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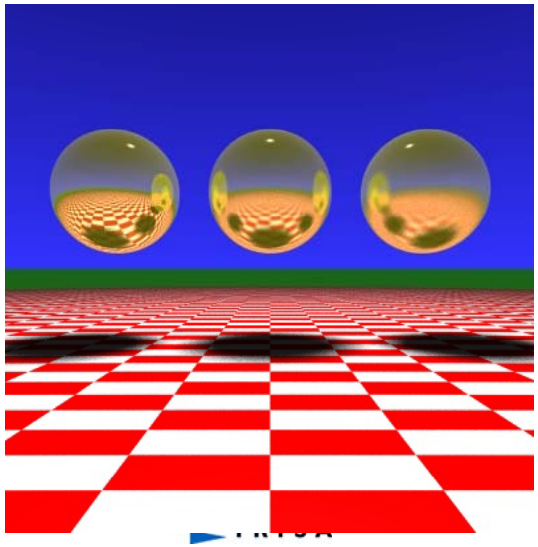
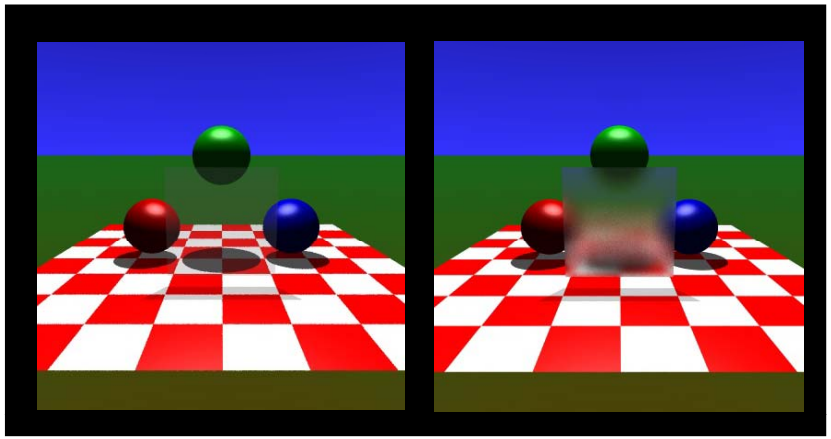
Translucency



Translucency



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Calculating the integrals

- How do we calculate these integrals?
  - Two-dimensional of the angles and ray-depth of the cone.
  - Unknown function -> the rendered scene.
- Use Monte-Carlo integration

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Shadows

- Ray tracing casts shadow from a point light source.
- Many light sources are illuminated over a finite area.
- The shadows between these are substantially different.
- Area light sources cast soft shadows
  - Penumbra
  - Umbra

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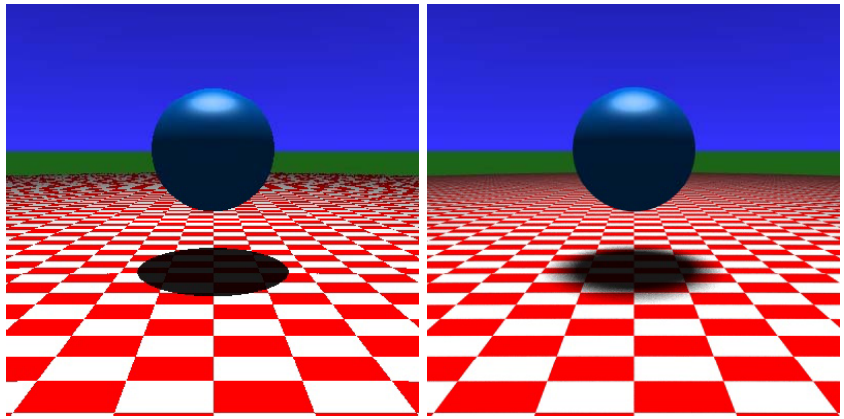
Soft Shadows



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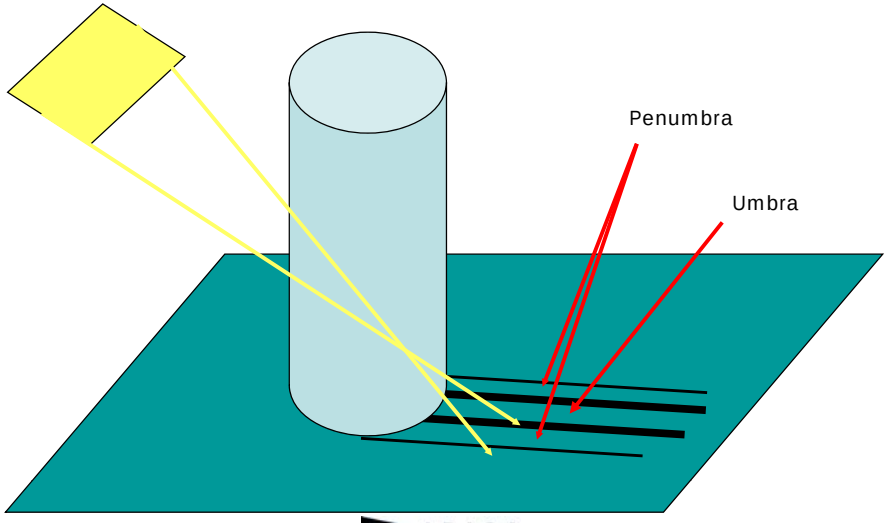
Soft Shadows



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Soft Shadows

- Umbra – No part of the light source is visible.
- Penumbra – Part of the light source is occluded and part is visible (to a varying degree).
- Which part? How much? What is the Light Intensity reaching the surface?

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Pros and Cons of Ray Tracing

- Advantages of ray tracing
  - All the advantages of the Phong model
  - Also handles shadows, reflection, and refraction
- Disadvantages of ray tracing
  - Computational expense
  - No diffuse inter-reflection between surfaces
  - Not physically accurate
- Other techniques exist to handle these shortcomings, at even greater expense!

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An Aside on Antialiasing

- Our simple ray tracer produces images with noticeable “jaggies”
- Jaggies and other unwanted artifacts can be eliminated by antialiasing:
  - Cast multiple rays through each image pixel
  - Color the pixel with the average ray contribution
  - An easy solution, but it increases the number of rays, and hence computation time, by an order of magnitude or more

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Intersection

Principle

- The scene is supposed to be expressed in the world coordinate system (WCS).
- It may be: A set of independent objects
- Purpose: intersect a scene with a ray whose equation is given by :
- $P = P_0 + t \, . \, D$
- where :

  P_0 is the ray origin;

  D = (dx, dy, dz) is the direction vector of the ray ;

  $t > 0$
- Intersection result = { ti / ti is a value of t corresponding to an intersection point }.
- Only the closest point to the ray origin is used to compute shading and secondary shot rays.

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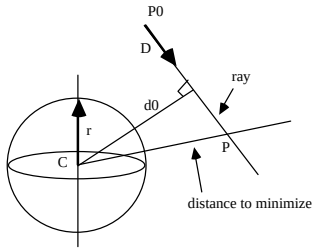
Intersection

Sphere

- d_0: Orthogonal distance between the ray and the center of the sphere of radius r and center C
- $P = P_0 + t \, . \, D$: the ray equation
- $P_0 = (X_0, Y_0, Z_0)$ D=(dx,dy,dz)
- If $d_0^2 <= r^2$, then the ray intersects the sphere
- Intersection points = solutions of $|| P_0 - C ||^2 + 2t \, . \, (P_0 - C) \bullet D + t^2 \, . \, || D ||^2 = r^2$
- d_0 is evaluated by minimizing the distance d between C and a point P on the ray.
- This gives: $d^2 = || P_0 + t \, . \, D - C ||^2 = || P_0 - C ||^2 + 2t \, . \, (P_0 - C) \bullet D + t^2 \, . \, || D ||^2$
- By setting to 0 the derivative of d^2 , we obtain : $t = ((P_0 - C) \bullet D \; / || D ||^2) = - (P_0 - C) \bullet D$
- After substitution : $d_0^2 = || P_0 - C ||^2 – ((P - C) \bullet D)^2$



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Intersection

Axis-aligned Parallelepiped

- Faces: perpendicular to the axes of the world coordinate system.
- First, the intersections between the ray and the faces x = x1 and x = x2 are computed.
- Two values of t are then obtained
- t1 = (x1 - x0) / dx and t2 = (x2 - x0) / dx.
- Interval: [Ix, Mx] = [min(t1, t2), max(t1, t2)]
- Same processing applied to the faces perpendicular to the y and z axes. Two other intervals: [Iy, My] and [Iz, Mz]
- The result is then an intersection interval given by :
  [I, M] = [max(Ix, Iy, Iz), min(Mx, My, Mz)]
- If I <= M then the ray intersects the parallelepiped bounding volume, otherwise it does not intersect it
- Closest intersection point: t=I

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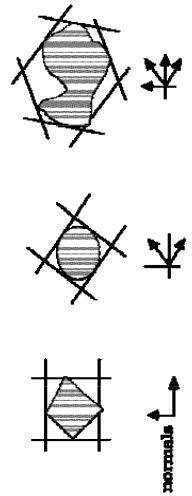
Intersection

Polyhedron

- Polyhedron = set of pairs of parallel faces
- Ni: normal to a pair of faces
- A pair of parallel faces is called slab



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Intersection

Polyhedron

- The intersection test is similar to that of a parallelepiped, except that the faces are not perpendicular to the axes of the coordinate system
- For each pair i, compute interval $[I_i, M_i]$
- Let N be the normal to a face
- N ● P + d = 0 the equation of the plane containing the face.

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Intersection

Polyhedron

- The value of t corresponding to the intersection between the ray and this face is computed by substituting the ray equation into that of the plane :
  t = - (d + N ● P0) / N ● D
- For each slab i , N=Ni and ===→

$$t = \alpha_i * d + \beta i$$

$$\alpha i = \frac{-1}{Ni \bullet D}$$

$$\beta i = \frac{-Ni \bullet P0}{Ni \bullet D}$$

- Given a slab i, these values are the same for all the polyhedra used as object bounding volumes

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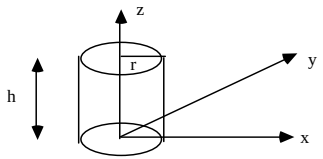
Intersection

Cylinder

- The cylinder : intersection between an infinite height cylinder and the subspace delimited by two planes which equations are
  z = 0 and z= h
- The intersection between the ray an the infinite height cylinder is first performed. This yields a first interval [t1,t2]
- The intersection with the two planes gives a second interval [t3, t4].
- The final intersection interval [I, M] results from the combination of these two intervals (as for the parallelepiped).



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Intersection

Cylinder: continued

- obtaining [t1, t2]
  - The equation of the infinite height cylinder :
  - $x^2 + y^2 = r^2$
  - Substituting the ray equation in this equation we obtain:
    $t^2 . (dx^2 + dy^2) + 2t . (x_0 . dx + y_0 . dy) + (x_0^2 + y_0^2 - r2) = 0$
  - Solving this equation gives the interval [t1, t2].
- *obtaining [t3, t4]*
  - Let A and B the two values of t resulting from the intersection with the two planes :
    $A = - z_0 / dz$ and $B = (h - z_0) / dz$
  - We get :
    t3 = min(A,B) and t4 = max(A, B)

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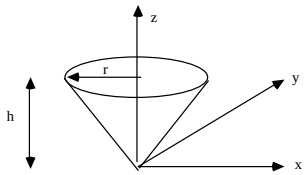
Intersection

Cone

- Intersection: performed in the LCS of the cone
- Cone: intersection between an infinite height cone and the subspace delimited by two planes, the equations of which are z = 0 and z = h.
- Intersection between the ray and the infinite height cone is first performed.
- The equation of this cone is given by :
  h2 . (x2 + y2) - r2 . z2 = 0.



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Intersection

Cone

- Substituting the ray equation in this equation yields an interval [t1, t2].
- Then the planes are in their turn intersected to give a second interval [t3, t4] such that :
  t3 = min(A, B) and t4 = max(A, B)
- where A = - z0 / dz and B = (h - z0) / dz.
- The final interval is the combination of these two intervals (as for the cylinder)

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Intersection

Polygon

- Several ray-polygon intersection methods have been proposed in the literature.
- Only two of them are presented .
- For all these methods, the intersection process consists of two steps :
  - *First step: Ray-Plane intersection test*
    - the goal of the first step is to perform the intersection between the ray and the plane containing the polygon
  - *Second step: Inside - Outside test*
    - the second step tests if the resulting point is inside or outside the polygon.

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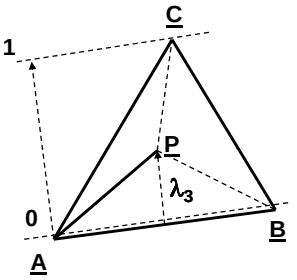
Intersection - Triangle

- Barycentric coordinates
  - Non-degenerate triangle ABC
    $\underline{P} = \lambda_1\underline{A} + \lambda_2\underline{B} + \lambda_3\underline{C}$
  - $\lambda_1 + \lambda_2 + \lambda_3 = 1$
  - $\lambda_i >= 0$
  - λ_3 = area(APB) / area(ACB),
  - λ_2 = area(APC) / area(ACB),
  - λ_1 = area(CPB) / area(ACB),
  - Area(APB)= $\frac{1}{2}\det(P\vec{A}, P\vec{B}) = \frac{1}{2}\left|P\vec{A}\times P\vec{B}\right| = \frac{1}{2}\left|P\vec{A}\right|\left|P\vec{B}\right|\sin(\hat{P})$



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Intersection

- **Polygon: Snyder's method**

- Ray-triangle intersection: extension to a polygon.
- Let Pi be the vertices of a triangle and Ni the associated normals which are used for normal interpolation across the triangle.
- Normal to the triangle: N = (P1 - P0) x (P2 - P0)
- A point P lying on the triangle plane satisfies :
  P ● N + d = 0 where d = - P0 ● N.
- To intersect a ray P = O + t . D with a triangle, first compute the t parameter of the intersection between the ray and the triangle plane
  t = (d - N ● O) / N ● D.

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Intersection

Polygon: Snyder's method

- Projecting the triangle into any other plane, except one that is orthogonal to the triangles plane will not change the barycentric coordinates of the triangle.
- This allows to simplify computations, since we can choose any of the coordinate system's three axis-aligned planes to project our triangle, thus throwing away one of the three coordinates and reducing the barycentric equations to R^2.
- For reasons of numerical stability we want to choose the dominant axis of the triangles normal for the projection.
- An index i0 is computed: equal either to 0 if | Nx | is maximum (i.e. the x axis is dominant) or to 1 if | Ny | is maximum or to 2 if | Nz | is maximum.

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Intersection

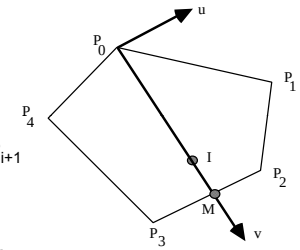
Polygon: Snyder's method

- Let i1 and i2 (i1, i2 ∈{0, 1, 2}) be two unequal indices different from i0. Compute the i1 and i2 components of the intersection point I:
  $Ii1 = Oi1 + t \cdot Di1$ and $Ii2 = Oi2 + t \cdot Di2$
- The inside-outside test can be performed by computing scalars ß0, ß1 and ß2 according to :
  $ßi = [(Pi+2 - Pi+1) \times (I - Pi+1)]i0 \ / \ [N]i0$
- The ßi are the barycentric coordinates of the point where the ray intersects the triangle plane.
- I is inside the triangle if and only if $0 \le ß \le 1$ for i {0, 1, 2}.
- The interpolated normal at point I is given by :
  $N' = ß0 \cdot N0 + ß1 \cdot N1 + ß2 \cdot N2.$
- Snyder's method can be easily extended up to polygons.
- The main idea is to consider a polygon as a union of triangles.

Intersection

Marchal's method

- I is the ray-plane intersection point.
- The Pi are transformed to the two dimensional coordinates system (u, v) whose origin is vertex P_0.
- The plane of this coordinates system is the polygon plane.
- The inside-outside test determines if an edge P_iP_{i+1} intersects the v axis at a point M (this may occur when the u components of P_i and P_{i+1} have different signs).
- If so, and if $P_0I < P_0M$ then I is inside the polygon, else it is outside.
- On the other hand, if none of the edges intersect the v axis, then I lies outside the polygon.



Intersection

Marchal's method

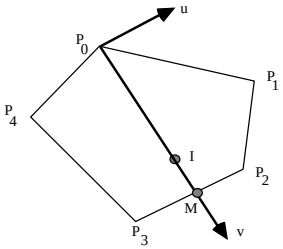
- The interpolated normal at point I is given by :

  $N_I = (P_0I / P_0M) \bullet N_M + (1 - P_0I / P_0M) \bullet N_0$

- where the normal N_M at point M is given by :

  $N_M = (P_iM / P_iP_{i+1}) \bullet N_{i+1} + (1 – P_iM/ P_iP_{i+1}) \cdot N_i$

and N_i, N_{i+1} are the normals at point P_i and P_{i+1}. P_iP_{i+1} is the intersected edge.

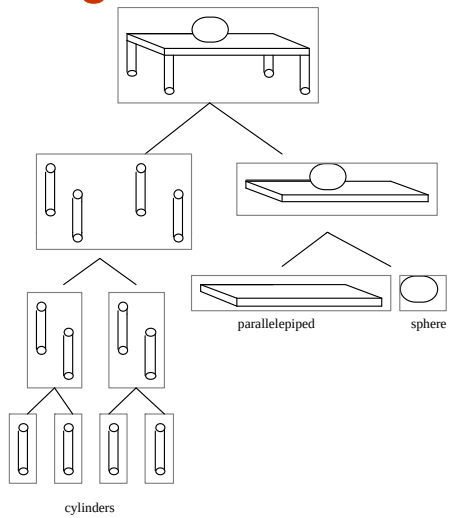


Bounding box

- To reduce the amount of ray-object intersections, its is absolutely necessary to use a hierarchical data structure .
- This data structure is a tree of bounding volumes.
- Bounding volumes are simple geometric objects which fit around the objects.
- They are chosen to be simple to intersect with a ray, such as spheres or parallelepipeds that have faces perpendicular to the axes.

Bounding box

- Example of a hierarchy of bounding boxes : binary tree.



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Bounding Volume

Different kinds of bounding Volume

- **Parallelepiped**
  - For the sake of speed up, the faces of this bounding volume are perpendicular to the axes of the World Coordinates System.
  - Its perspective projection onto the screen plane is often used to filter the primary rays (rays starting at the eye location).
- **Sphere and Ellipsoid**
  - They may be used to filter the reflected and refracted rays and those directed to the light sources.
- **Polyhedron**
  - Intersection of slabs: a slab is a pair of parallel faces

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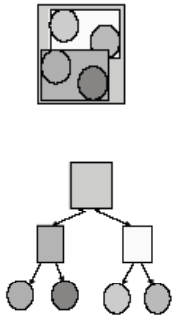
Bounding Volume Hierarchy

- Organize objects into a tree
- Group objects in the tree
  - based on spatial relationships
- Each node in the tree contains a bounding box of all the objects below it



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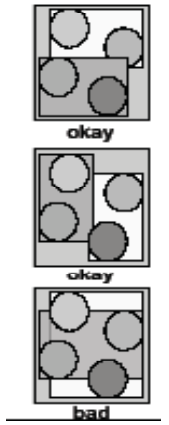
Bounding Volume Hierarchy (BVH)

- Determining optimal BVH structure is NP-hard problem
- Heuristic approaches:
  - Cost models (minimize volume or surface area)
  - Spatial models
- Categories of approaches:
  - Top down
  - Bottom up



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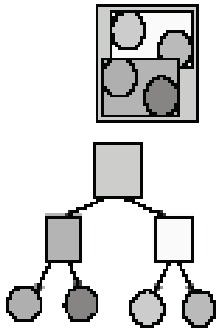
Median Cut BVH Construction

Top down approach:

- Sort objects by position on axis
  - cycle through x,y,z
  - use center of bounding box
- Insert tree node with half of objects on left and half on right



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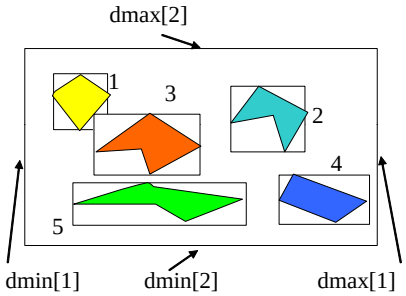
Median Cut BVH Construction

1. L= {list of bounding volume numbers}
2. Choose widest slab:
   dmax[2] – dmin[2] or dmax[1] – dmin[1]
   (In this example :
   max width = dmax[1] – dmin[1]
3. Then choose slab of max width
4. Sort the bounding volumes wrp to increasing dmin[number_of_widest_slab]
5. We get a sorted list L = {1,5,3,2,4}
6. Split L into two sub-lists L1 and L2
7. We get : L1 = {1,5,3} L2 = {2,4}
8. Go to 1 with L = L1 then L = L2



Leaf = one or more objects

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Bottom up BVH Construction

- Add objects one at a time to tree
- Insert to subtree that would cause smallest increase to area

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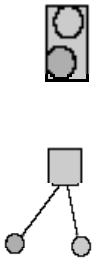
Bottom up BVH Construction

- Add objects one at a time to tree
- Insert to subtree that would cause smallest increase to area



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Bottom up BVH Construction

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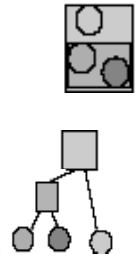

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Bottom up BVH Construction

- Add objects one at a time to tree
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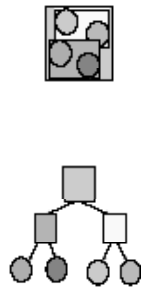

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Intersection Test Using the BVH

- Once the hierarchy of bounding volumes has been built, the ray-scene intersection test is performed as follows.
  - The hierarchy is searched from the root to the leaves.
  - During this search, at a node N, the associated bounding volume is checked for an intersection with the current ray.
  - If the bounding volume of N is intersected, those of its children node are in their turn checked for an intersection.
- This process is repeated recursively and ends up at the leaf nodes.
- Else, if the bounding volume of N is not intersected by the ray, the associated subtree is left out, that is, it is not searched, which saves time.

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Spatial Subdivision

- The rectangular bounding volume of the scene is subdivided into 3D cells
- Each cell contains a few objects of the scene
- When a ray enters a cell, we check the objects within this cell for an intersection with the ray
- If the intersection process ends up with success then no need to check the rest of the objects
- If the ray fails to hit any object in the cell then it moves to the next 3D cell
- Repeat the process until intersection is found

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Spatial Subdivision

- **Two procedures**
  - A procedure which performs a spatial subdivision of the scene into 3D cells, each of them containing a small portion of the database
  - A second procedure which determines the next cell along a ray

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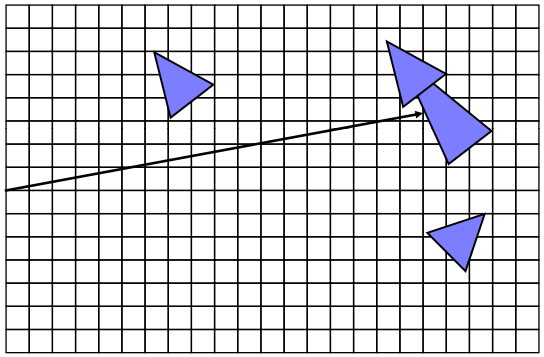
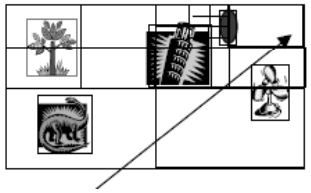
Spatial Subdivision

- **Two procedures**



Uniform Grid



Non uniform Grid

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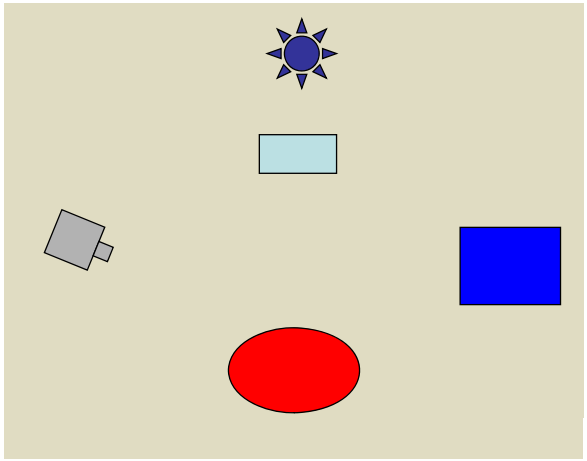
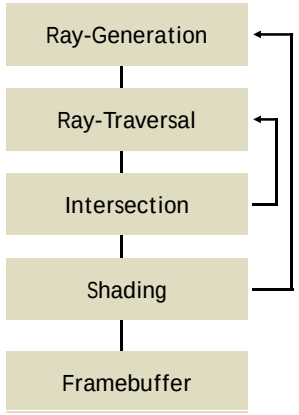
What is Ray Tracing in a subdivided space?





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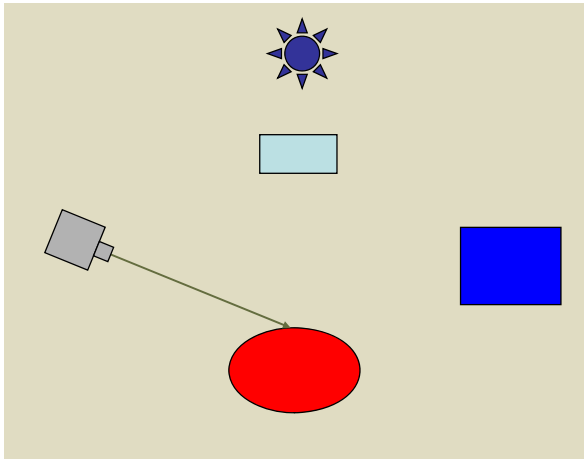

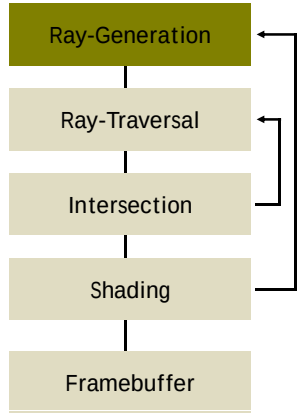

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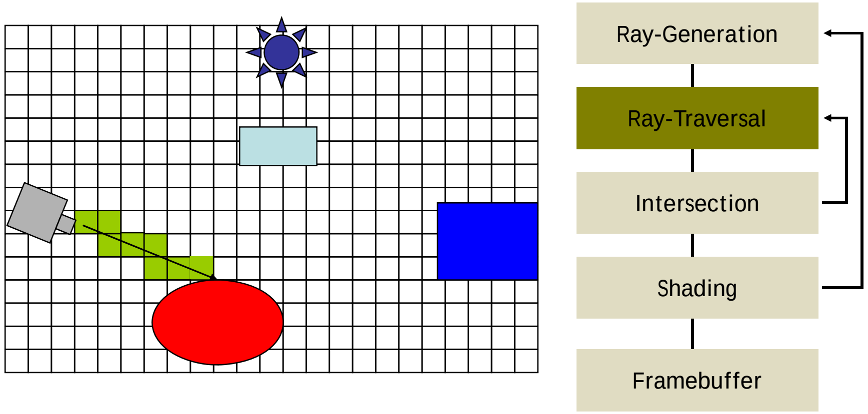
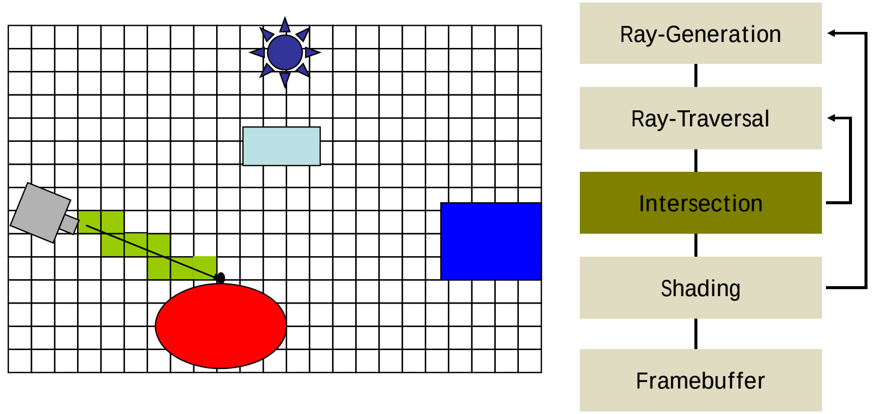
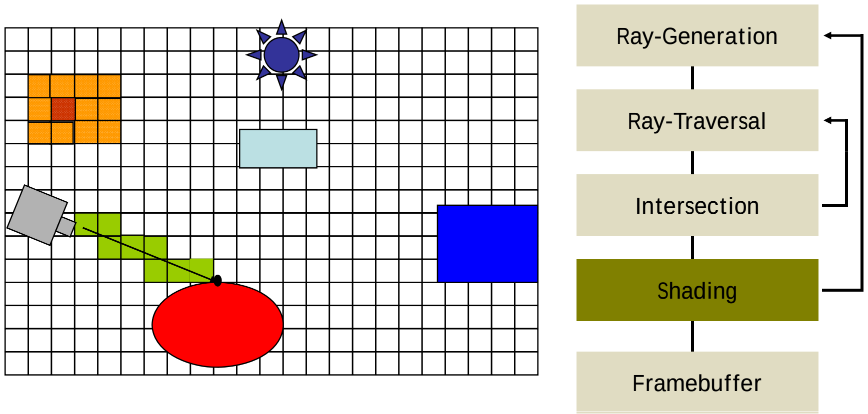


What is Ray Tracing in a subdivided space?



What is Ray Tracing in a subdivided space?



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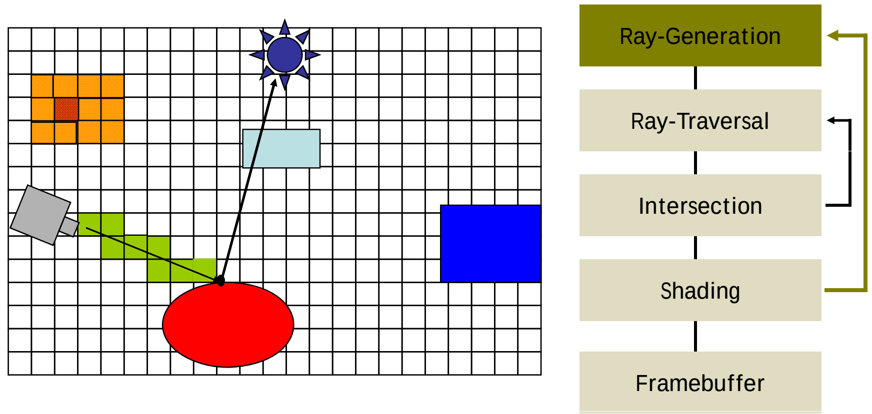
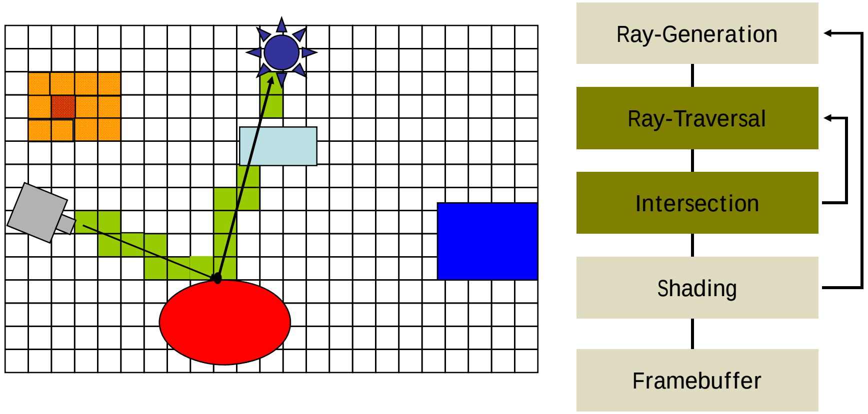
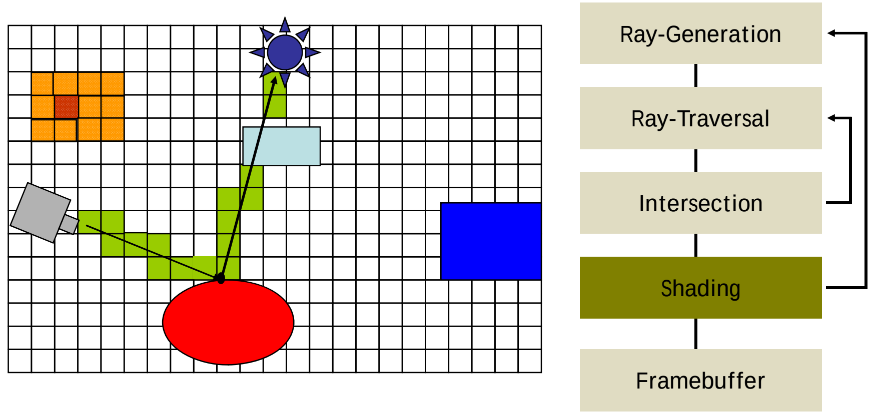


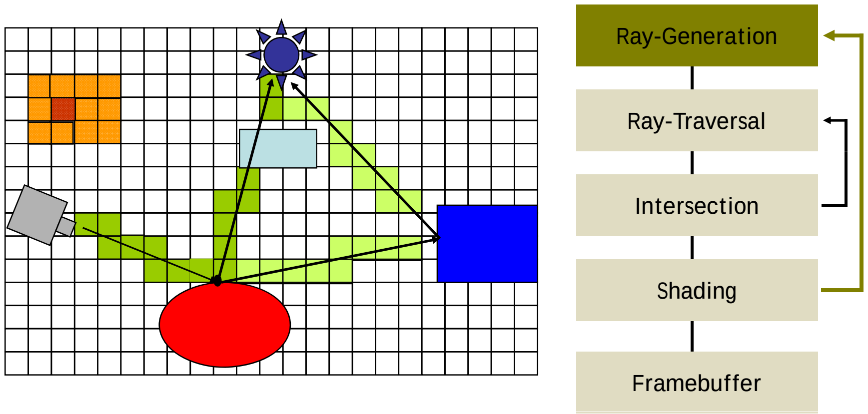
What is Ray Tracing in a subdivided space?



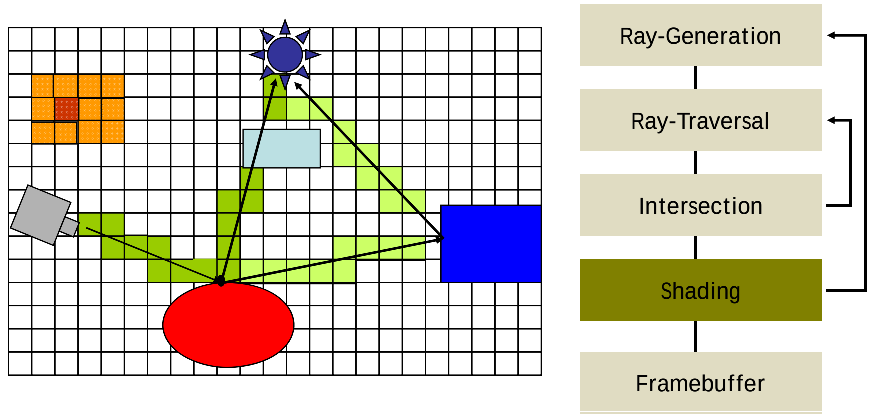
What is Ray Tracing in a subdivided space?



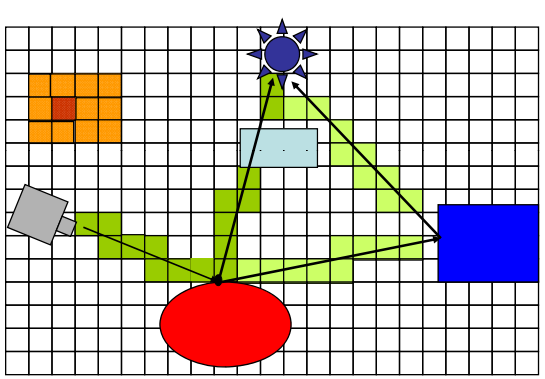
What is Ray Tracing in a subdivided space?

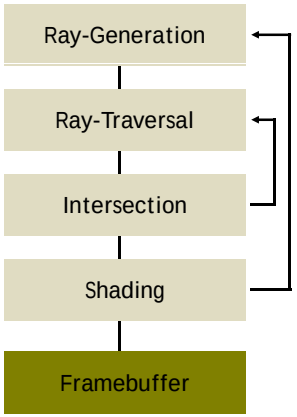


What is Ray Tracing in a subdivided space?



What is Ray Tracing in a subdivided space?





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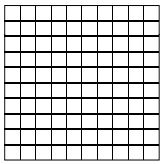
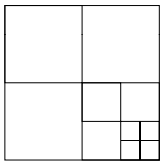
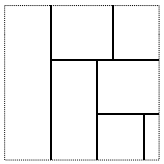
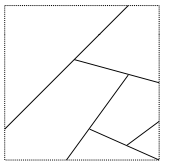
Spatial Subdivision

- Different kinds of subdivision


Uniform Spatial Sub


Quadtree/Octree


kd-tree


BSP-tree

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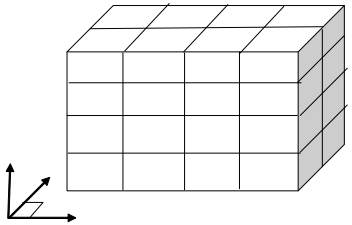
Uniform Grid

- The rectangular bounding volume of the scene is subdivided into a uniform 3D grid of rectangular cells
- The grid is represented by a 3D array, the indices of which are i, j and k corresponding to the x, y and z axes respectively
- Each cell is represented by a data structure containing a pointer to the objects partially or totally within the cell



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Uniform Grid

Ray Traversal Algorithm: Classical Method

- Let G[i][j][k] be the 3D array representing the 3D grid
- Let P the point where the ray leaves the current cell and D the ray direction
- P is the outgoing point
- Let w be the axis perpendicular to the face which contains P
- Let u (x, y or z) be the index (i, j or k) of the current cell corresponding to w
- If Dw > 0 then the index u of the next cell is u = u + 1, the other indices are unchanged
- Else it is : u = u – 1
- Example :
- If w = z then u = k
- If Dz > 0 then the index of the next cell along the ray is k = k + 1, while the other indices do not change
- If the current cell is G[i][j][k] then the next cell along the ray is G[i][j][k + 1] if Dz > 0, or G[i][j][k - 1] if Dz < 0

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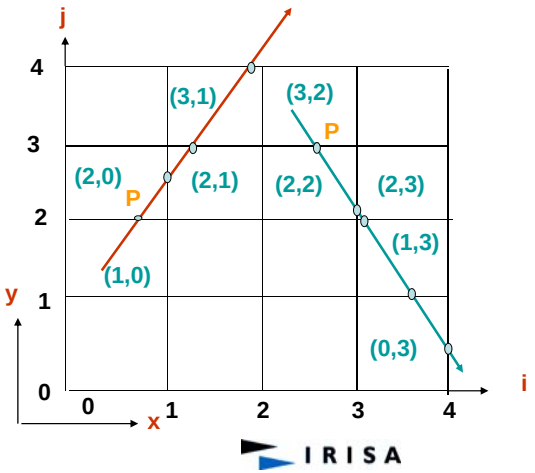
Uniform Grid

Ray Traversal Algorithm: Classical Method



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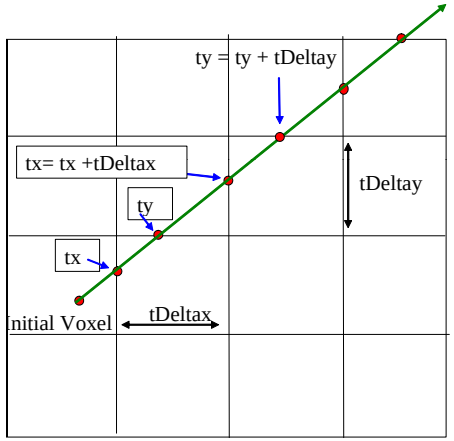
Uniform Grid

- **Ray Traversal Algorithm: Amanatide's Algorithm**



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Uniform Grid

Ray Traversal Algorithm: Amanatide's Algorithm

Initialization

- Ray equation : P = P0 + t . D
- Identify the voxel containing the ray origin O
- If O is outside the grid, find the point through which the ray enters the grid and determine the adjacent voxel
- X, Y and Z : voxel indices
- StepX, stepY and stepZ : initialized to 1, incremented or decremented as the ray crosses the voxel boundaries
- tx, ty and tz : values of t corresponding to the points resulting from the intersection between the ray and 3 faces of the initial voxel
- tDeltaX, tDeltaY and tDeltaZ : distance travelled by the ray between two successive faces perpendicular to the x, y and z faces respectively

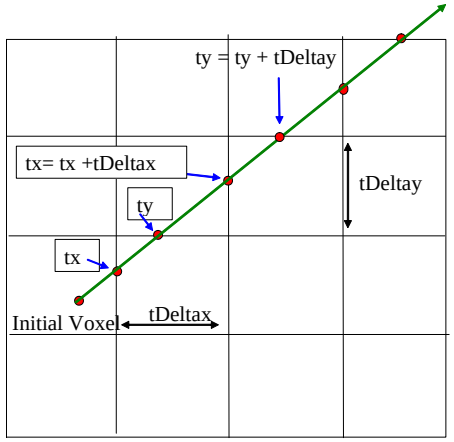
IRISA

Uniform Grid

Ray Traversal Algorithm: Amanatide's Algorithm

Algorithm

```
Min = min(tx,ty,tz) ;
switch(Min)
{
    case tx :
            X += stepX ;
            tx += tDeltax ;
            break ;
    case ty
            Y += stepY ;
            ty += tDeltay ;
            break ;
    case ty
            Z += stepZ ;
            tz += tDeltaz ;
            break ;
}
```



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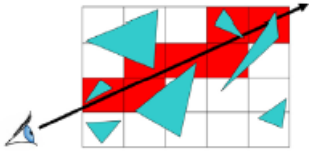
Uniform Grid

- Advantages?
  - easy to construct
  - easy to traverse
- Disadvantages?
  - may be only sparsely filled
  - geometry may still be clumped (say, densely grouped)



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Non Uniform Grid

- The rectangular bounding volume of the scene is recursively sliced :
  - either simultaneously by 3 planes perpendicular to the x, y and z axes: Octree
  - or by one plane at a time perpendicular to an axis: Kd-tree, Bsp tree
  - or by one plane at a time non necessary perpendicular to an axis: Bsp tree
- Each slicing plane divides a space (a 3D cell) into two subspaces (3D cells)
- The subdivision process stops either when a cell contains partially or totally a minimum number of objects, or the maximum subdivision level is reached for each axis
- The result is a linear array of rectangular cells or a binary tree or an octree
- Each cell is represented by a data structure containing a pointer to the objects partially or totally within it

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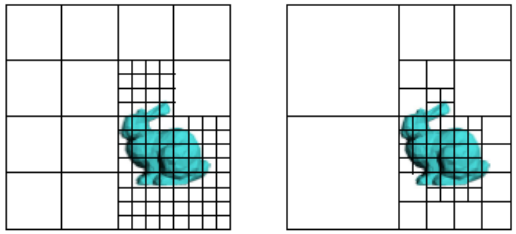
Non Uniform Grid

- Subdivide until each cell contains no more than *n* elements, or maximum depth *d* is reached



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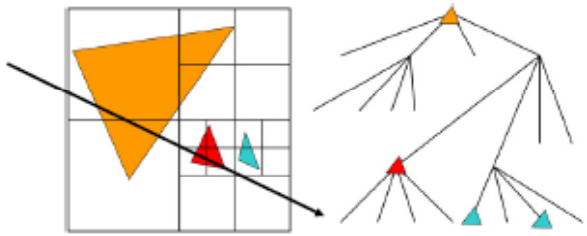
Non Uniform Grid

- Advantages?
  - grid complexity matches geometric density
- Disadvantages?
  - more expensive to traverse (especially octree)



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Non Uniform Grid: Kd-Tree

- Subdivide only 1 dimension
- Do not subdivide at the center
- Which axis to pick?
- What point on the axis to pick?
- One heuristic:
  - Sort objects on each axis
  - Pick point corresponding to “middle” object
  - Pick axis that has “best” distribution of objects
  - L = n/2, R = n/2 (ideal), where L→Left and R→Right
  - Realistically,
    - minimize (L-R) and
    - L approx. n/2, R approx. n/2

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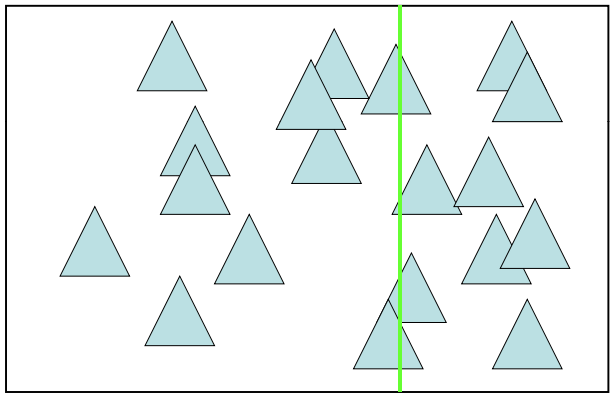
kD-Trees



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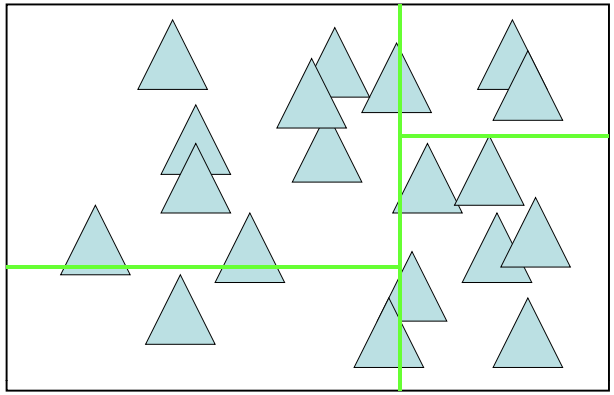
kD-Trees



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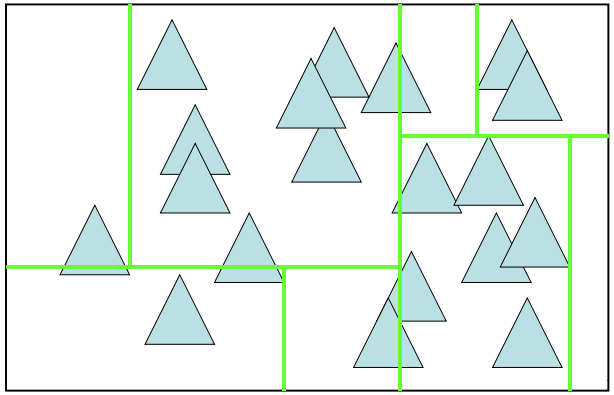
kD-Trees



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kD-Trees: Data Structure

```
Struct KdTreeNode {
  int   axis;    // Both, x or y or z split plane (0,1,2), 3 for leaf
  float   value;      // Interior, split position x, y or z
  int  nPrims;  // Leaf
  Bounding_Box  bounds;
 KdTreeNode   *LeftChild;          // interior
 KdTreeNode   *RightChild;        // interior
}
```

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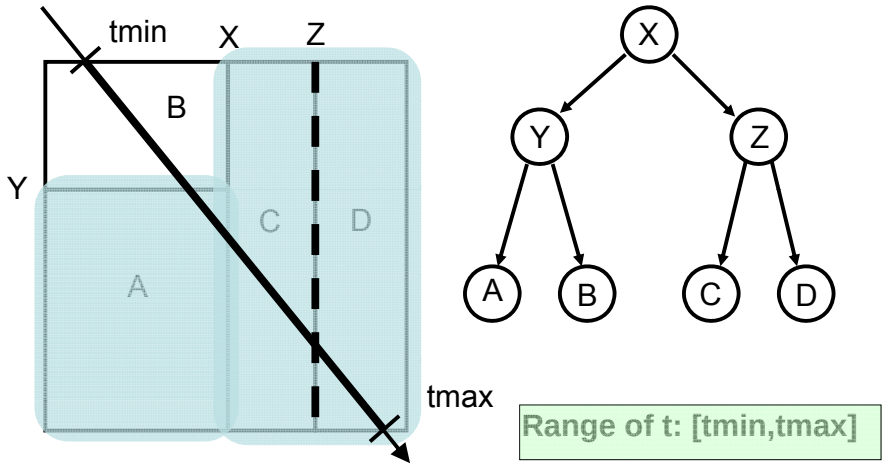
KD-Tree: Traversal



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KD-Tree: Traversal

- Input: a tree and a ray
- Search for the first intersected primitive in the tree
- Traversal: start from the root
- Use of a stack
- First range of t, [tmin,tmax]: associated with the scene bounding box
- Internal node encountered: ray is classified wrp to the splitting plane
  - If range lies entirely in one side of the plane, traversal moves to the appropriate child
  - If the range straddles the plane, traversal will continue to the first child hit by the ray while the second child is pushed onto the stack along with its range [tmin,tmax]
- Traversal proceeds down the tree, occasionally pushing items onto the stack, until a leaf node is reached.

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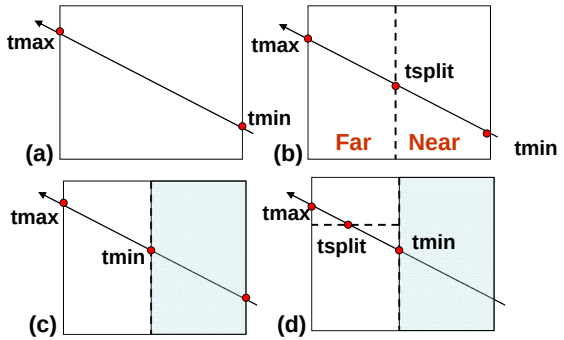
KD-Tree: Traversal



- (d) Sequence continues, processing tree nodes in depth first, front-to-back traversal, until closest intersection is found or the ray exists the tree

- (a) Initial parametric range [tmin, tmax] : intersection ray-bounding box
- (b) The ray first enters the child “near” which has [tmin,tsplit] as parametric range. If leaf then intersection, otherwise child nodes are processed
- (c) If no hit or a hit beyond [tmin,tsplit] then “far node” is processed

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KD-Tree: Traversal

```
kd-search( tree, ray )
        (global-tmin, global-tmax) = intersect( tree.bounds, ray )
{
search-node( tree.root, ray, global-tmin, global-tmax )
}

search-node( node, ray, tmin, tmax )
{
      if( node.is-leaf )
          search-leaf( node, ray, tmin, tmax )
      else
          search-split( node, ray, tmin, tmax )
}
```

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KD-Tree: Traversal

```
search-split( split, ray, tmin, tmax ) {
  a = split.axis
  thit = ( split.value - ray.origin[a] ) / ray.direction[a]
  (first, second) = order( ray.direction[a], split.left,split.right )
  if( thit >= tmax or thit < 0 )
    search-node( first, ray, tmin, tmax )
  else if( thit <= tmin )
          search-node( second, ray, tmin, tmax )
        else {
          stack.push( second, thit, tmax )
          search-node( first, ray, tmin, thit )
        }
}
```

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KD-Tree: Traversal

```
search-leaf( leaf, ray, tmin, tmax ) {
  // search for a hit in this leaf
  if( found-hit and hit.t < tmax )
      succeed( hit )
  else
      continue-search( leaf, ray, tmin, tmax )
}

continue-search( leaf, ray, tmin, tmax ){
  if( stack.is-empty )
      fail()
  else {
     (n, tmin, tmax) = stack.pop()
     search-node( n, ray, tmin, tmax )
  }
}
```

- **Remark**

If stack empty, then no intersection along the ray and the search terminates

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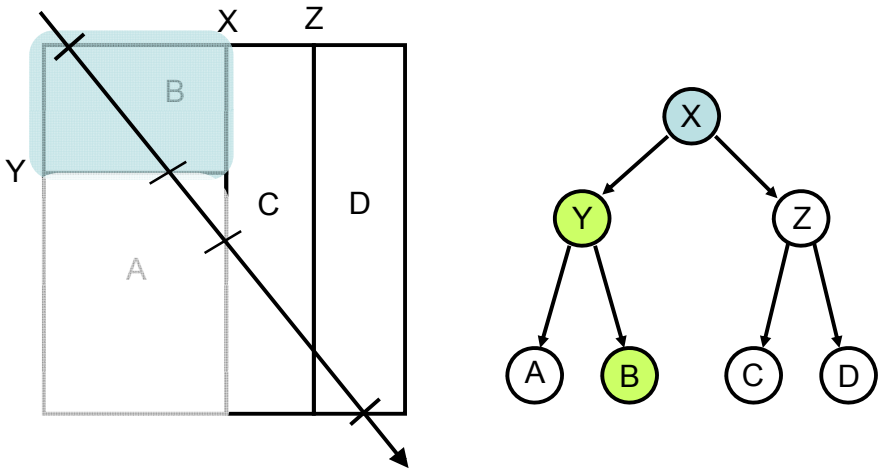
KD-Tree Traversal



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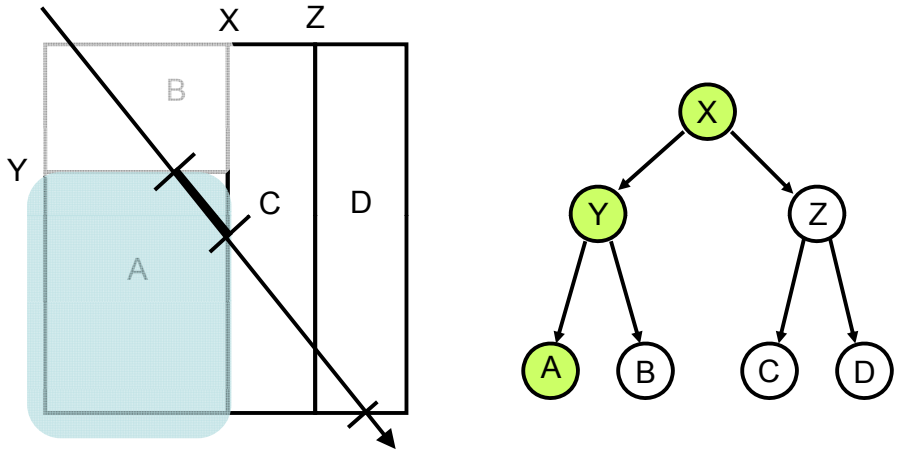
Kd-tree traversal: Observation



Current leaf's tmax = Next leaf's min

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Kd-tree traversal: Observation

- Eliminate stack operations
- How?
  - If the traversal reaches a leaf and fails to find a hit:
    - Restart the search at the root
    - With tmin advanced to the end of the leaf
    - The first leaf intersected by the modified range is the next leaf that needs to be traversed

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Kd-tree traversal: Restart

```
continue-search( leaf, ray, tmin, tmax )
{
  if( tmax == global-tmax )
     fail()
  else {
     tmin = tmax
     tmax = global-tmax
     search-node( tree.root, ray, tmin, tmax )
   }
}
```

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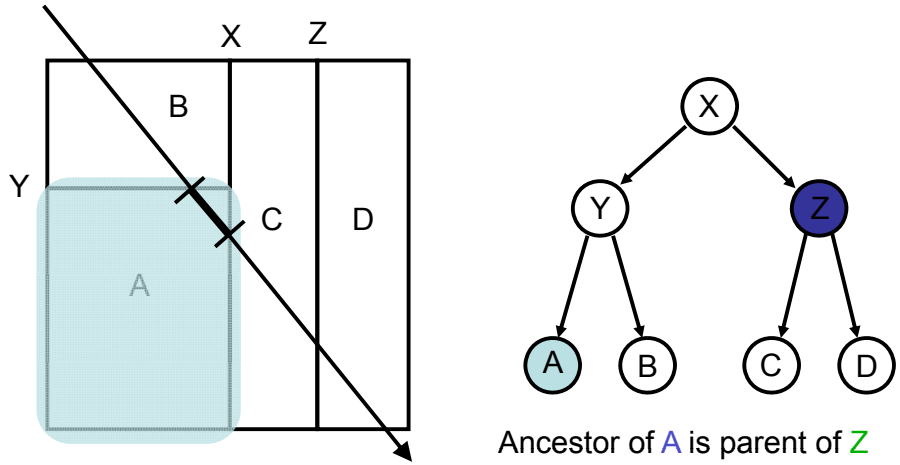
Observation



Ancestor of A is parent of Z

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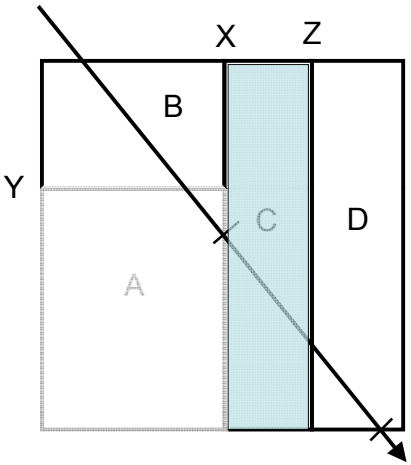
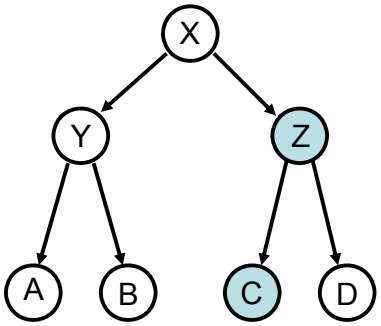
Kd-tree: Observation





Ancestor of A is parent of Z

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Kd-tree: Backtrack

- In the traditional, a node pushed onto the stack is always the other (second) child of one of the current node's ancestors
- Thus, possible to reach the parent of the node atop the stack by following a chain of parent links (which we can store in the nodes of the tree) from the current node.
- If we again employ the tactic of advancing *tmin* to the end of the last leaf visited, then we will be able to recognize the appropriate parent as the closest ancestor that has a nonempty intersection with the remaining (*tmin*; *tmax*) range.
- Bounding boxes are stored with internal nodes
- Parents links are stored in all nodes
- Increase per-node storage

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KD-Backtrack

```
continue-search( leaf, ray, tmin, tmax ) {
   if( tmax == global-tmax )
      fail()
   else  {
      tmin = tmax
      tmax = global-tmax
      backtrack( leaf.parent, ray, tmin, tmax )
    {
}

backtrack( split, ray, tmin, tmax ) {
   (t0,t1) = intersect( split.bounds, ray, tmin, tmax )
   if( no-intersection )
      backtrack( split.parent, ray, tmin, tmax )
   else
      search( split, ray, t0, t1 )
}
```

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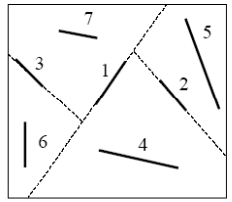
BSP Tree

- Generalization of kd-trees
- Splitting plane is not axis aligned
- Used in games: DOOM



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BSP tree

- A Binary Space Partitioning (BSP) tree data structure
  - Recursive, Hierarchical subdivision of *n-dimensional* space into convex subspaces.
- BSP tree construction
  - Partition a subspace by a hyper-plane that touches the edge of the subspace.
  - The result is two new subspaces that can be further partitioned by recursive application of the method.
- A "hyperplane" in an n-dimensional space is an n-1 dimensional object which can be used to divide the space into two half-spaces.

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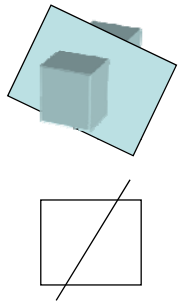
BSP tree

- example:
  - In three dimensional space, the "hyperplane" is a plane.
  - In two dimensional space, it is a line.
- BSP trees are extremely versatile, because they are powerful sorting and classification structures.
  - Hidden surface removal
  - Ray tracing hierarchies
  - Solid modeling
  - Robot motion planning.
- Intensive time and space preprocessing vs. linear display algorithm.



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Building a BSP tree

- Given a set of polygons in three dimensional space, we would like to build a BSP tree which contains all of the polygons.
- The algorithm to build a BSP tree:
  - Select a partition plane.
  - Partition the set of polygons with the plane.
  - Recurse with each of the two new sets.
- The choice of partition plane depends on how the tree will be used, and what sort of efficiency criteria you have for the construction.

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Building a BSP tree

- For some purposes, it is appropriate to choose the partition plane from the input set of polygons
  - Scan-conversion
- Other applications may benefit more from axis aligned orthogonal partitions
  - Ray tracing
  - Space subdivision.
- It is desirable to have a balanced tree, where each leaf contains roughly the same number of polygons.
- It is desirable to minimize polygon splitting.
  - Finding the optimal split is hard, we use a heuristic
    - Testing the plane against a small random number of (5-6) polygons for split.

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BSP tree: Partitioning

- Classify each member of the set with respect to the plane.
- If a polygon lies entirely on one side of the hyper-plane
  - It is added to the partition set for the proper side.
- If a polygon spans the plane – keep in the node
- If the polygon intersect the hyper-plane
  - Split it as needed and add the parts the proper sets.

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BSP tree: **When to stop?**

- The decision to terminate the tree construction is a matter of the specific application.
  - Some applications will benefit from termination when the number of polygons in a leaf node is below a maximum value.
  - Other methods continue until every polygon is placed in an internal node.
- Another criteria that can be used is the maximum tree depth.

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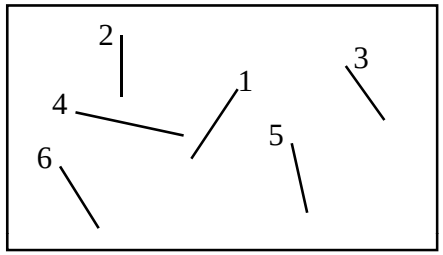
BSP tree: example

- One of the most important properties of BSP trees is that it is view independent.
- For example, consider the following case:



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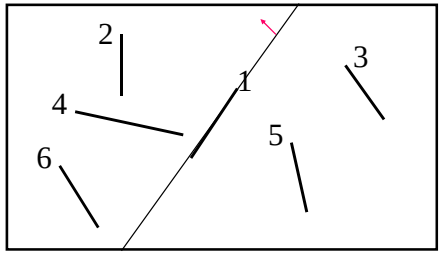
BSP tree: example

- Splitting the plane using the ordered lines from the input, we get the following:



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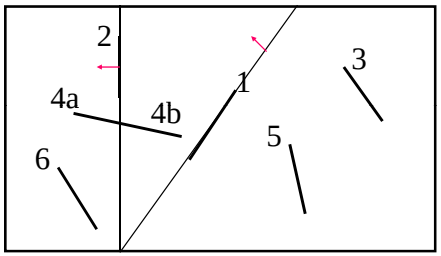
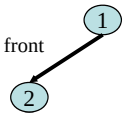
BSP tree: example





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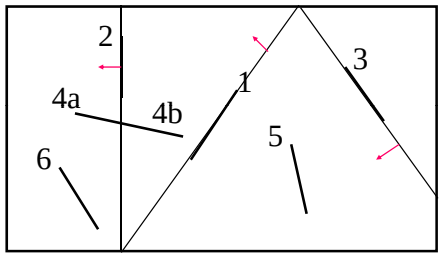
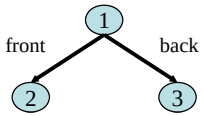
BSP tree: example





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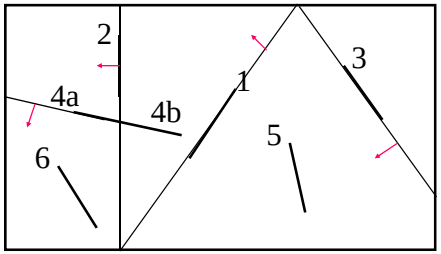
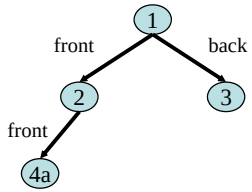
BSP tree: example





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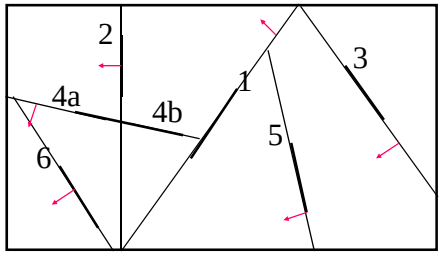
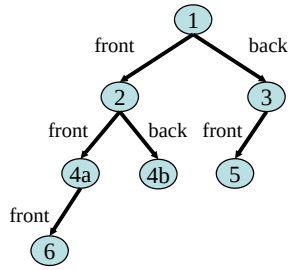
BSP tree: example





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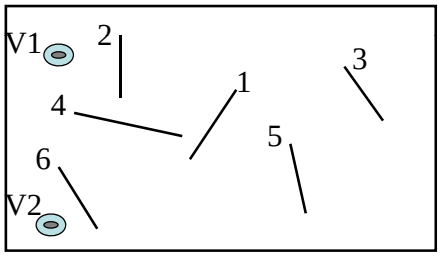
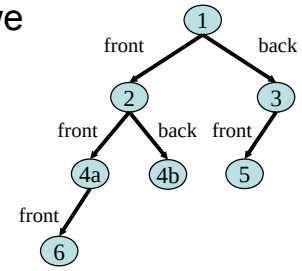
BSP tree: example

- Now, we can choose several view points, and choose the painting order according to the tree we have created.





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BSP tree: Hidden Surface Removal

- Probably the most common application of BSP trees is hidden surface removal in three dimension.
- BSP trees provide an elegant, efficient method for sorting polygons via a depth first tree walk. This fact can be exploited in a back to front "*painter's algorithm*".
- The idea behind the *painter's algorithm* is to draw polygons far away from the eye first, followed by drawing those that are close to the eye.
- Hidden surfaces will be written over in the image as the surfaces that obscure them are drawn.
- Can assist in 3D clipping.
- Can support Back Face Culling.

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BSP tree: Painting

- One reason that BSP trees are so elegant for the painter's algorithm is that the splitting of complex polygons is an automatic part of tree construction.
- When building a BSP tree specifically for hidden surface removal, the partition planes are usually chosen from the input polygon set.
- However, any arbitrary plane can be used if there are no intersecting or concave polygons.

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BSP tree: Drawing the scene

- To draw the contents of the tree:
  - Perform a back to front tree traversal.
  - Begin at the root node and classify the eye position with respect to the partition plane.
    - Draw the subtree at the far child from the eye
    - Draw the polygons in this node
    - Draw the near subtree.
  - Repeat this procedure recursively for each subtree.
- Front to back rendering is also possible.

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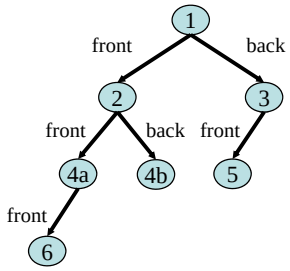
BSP tree: Hidden Surface Removal

- The painting order from V1:
  - 3, 5, 1, 4b, 2, 6, 4a
- The painting order from V2:
  - 3, 5, 1, 4b, 2, 4a, 6



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BSP tree: Ray Tracing

- Accelerating Ray Tracing
- Rectangular bounding volume of the scene: recursively subdivided
- Subdivision: Splitting planes are axis aligned
- Each splitting plane splits a cell into two equally sized sub-cells
- Choose x, y and z axis one at a time (alternate)

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Octree

- Useful for reducing the number of ray-object intersections.
- The bounded 3D world to be ray traced is subdivided into cells of varying size. Each cell contains a list of objects (of approximately the same length) which intersect it.
- Given a ray to be traced, a list of cells intersected by the ray is determined. Intersection calculations are performed only with these objects.
- Furthermore, if the cells may be accessed in the order of advance of the ray, the procedure may terminate once the first intersection is discovered.

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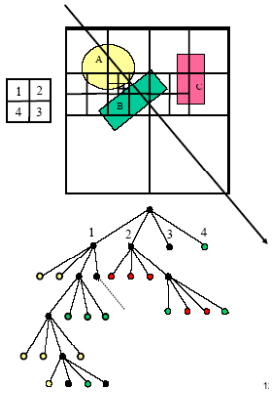
Octree

- Each node of the tree has eight children, corresponding to halving the space along all of the three axes.
- A node is a leaf if the subspace it represents intersects at most a given number of objects.
- The two basic operations needed for ray tracing octrees are:
  - Locating the leaf cell containing a given 3D point (point location).
  - Locating the next cell intersecting a given ray.
- The first is a standard octree traversal. The second is accomplished by repeating the first with a point along the ray just outside the current cell.



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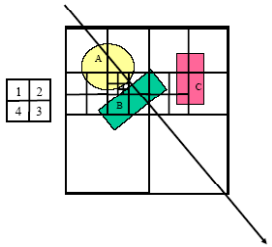
Octree

- Octrees ignore the *directionality* of objects.
- Subdivision is always in predefined directions and places.
- **Advantage:** Simple construction. Point location is easy.
- **Disadvantage:** Non-optimal subdivision (large trees).



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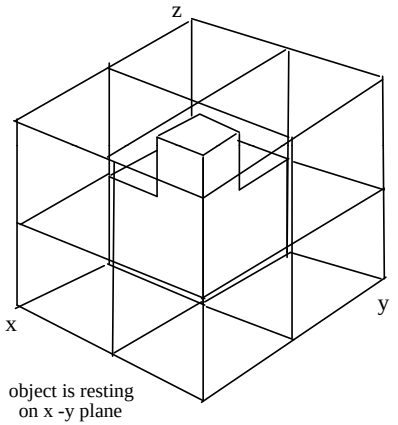
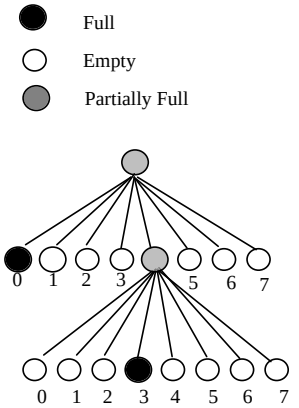
Octree: example





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Octree: traversal

1. Determine the first intersection point **F** between the ray and the scene's axis aligned bounding box (SAABB)
2. Push **F** along the normal to the face containing it,
3. Pushing consists in adding to the P's coordinates a value deltax (resp. deltay, deltaz) which is equal to half the length of the x side (resp. y, z) of the smallest cell.
4. Search for the cell (containing **F**) in the tree
5. If no intersection in the cell, compute outgoing point **P**
6. Push **P** along the normal to the cell's face containing it
7. The results is another point **P'**
8. Search for the cell (containing **P'**) in the tree
9. Go to 1 until intersection

Remark: If **P** is on an edge or a vertex of a cell, push it simultaneously in the directions of the normals to the faces sharing it

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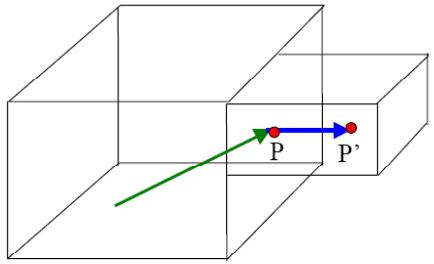
Octree: traversal



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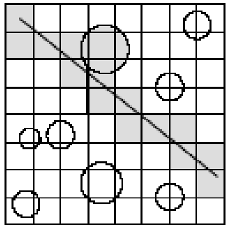
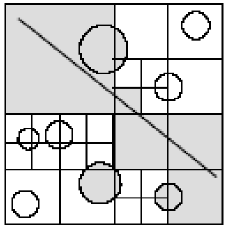
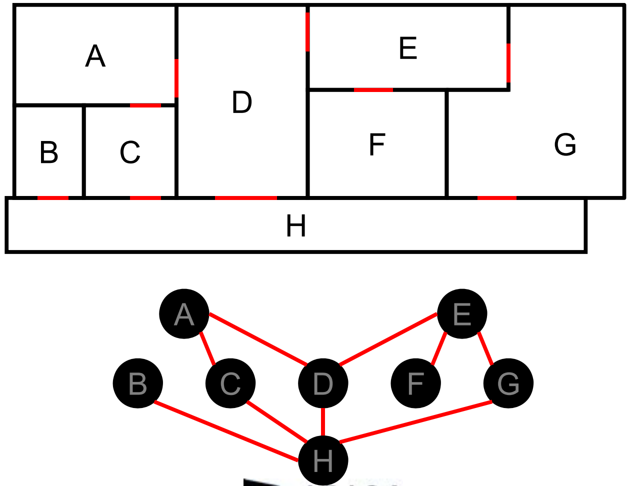
Uniform vs. Adaptive Subdivision

- Uniform: too much traversed empty cells
- Adaptive: less empty cells

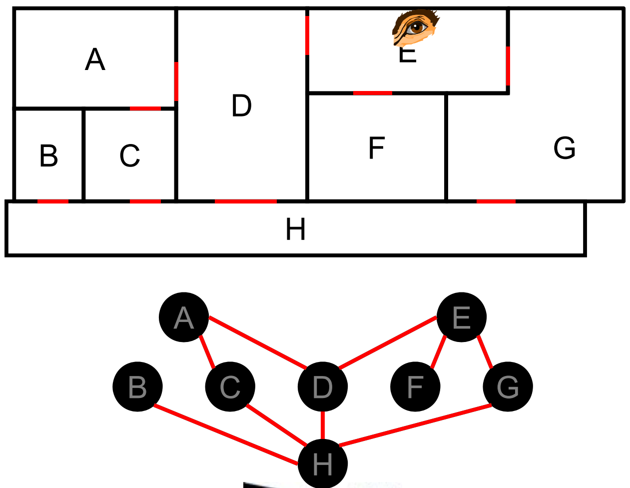




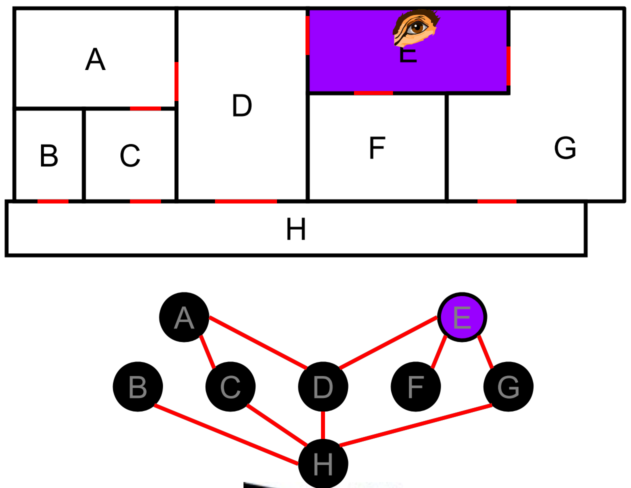
Cells & Portals



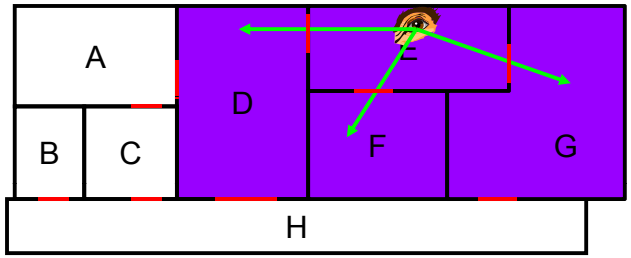
Cells & Portals

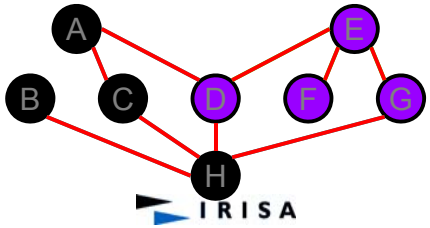


Cells & Portals



Cells & Portals





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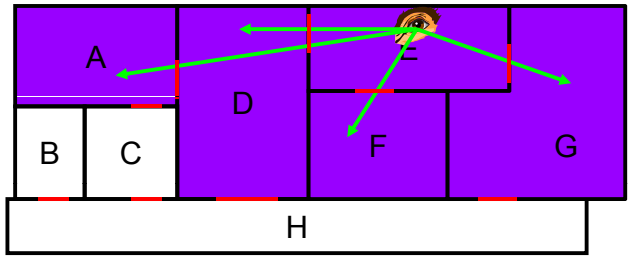
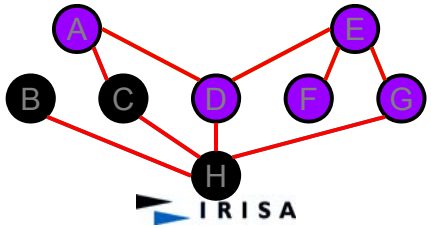
Cells & Portals





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Cells & Portals

Teller and Sequin's Approach

- Decompose space into convex cells: use walls as splitting polygons
- For each cell, identify its boundary edges into two sets: opaque or portal
- Precompute visibility among cells
- During viewing (eg, walkthrough phase), use the precomputed Potentially Visible polygon Set (PVS) of each cell to speed-up rendering

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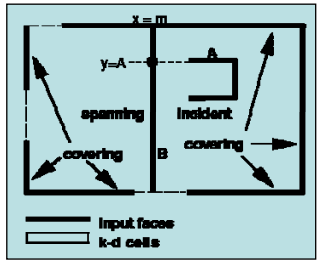
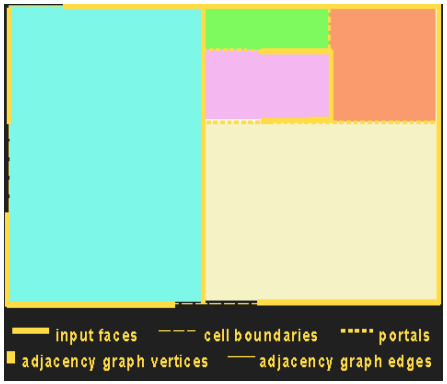
Cells & Portals: Space Subdivision

Input Scene:





Spatial subdivision: Generated by computing a k-d tree of the input faces

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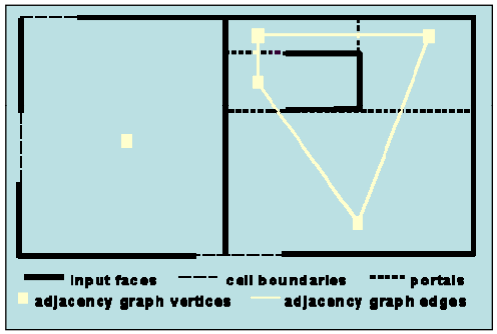
Determining Adjacent Information



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Finding Sightlines

Problem:

Given set of portals forming path from one portal to another, how do we determine if there is a sightline that passes through all those portals?

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Finding Sightlines

- **Key insight: orient portals**

- **All portal left end points must be on positive side of the line**

- **All portal right end points must be on negative side of the line**

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Finding Sightlines

- **Unknown line: S**
- **Constraints:**
  - **S . L >= 0 for all portal left points**
  - **S . R <= 0 for all portal right points**
- **This is a linear programming problem.**

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Cell to Cell Visibility

- **Given a cell and the sightline algorithm, we need to find, for each cell, all cells visible from it**
- **Use adjacency lists to traverse graph depth First**
- **At each cell, recurse only if sightline test is positive**

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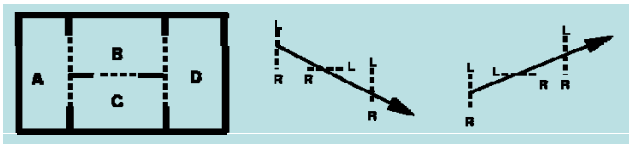
Computing the PVS of a cell



Linear programming problem:

$$S \cdot R \geq 0, \quad \forall L \in L$$
$$S \cdot R \leq 0, \quad \forall R \in R$$

```
Find_Visible_Cells(cell C, portal sequence P, visible cell set V)
  V=V ∪ C
  for each neighbor N of C
    for each portal p connecting C and N
      orient p from C to N
      P' = P concatenate p
      if Stabbing_Line(P') exists then
        Find_Visible_Cells (N, P', V)
```

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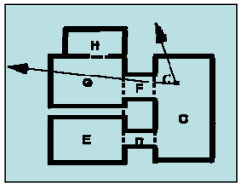
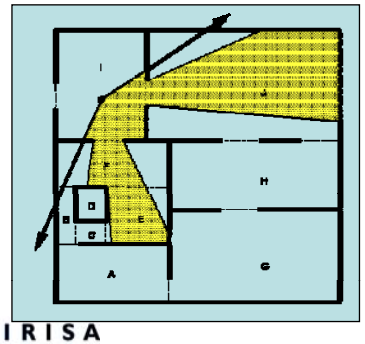
Eye-to-Cell Visibility

The eye-to-cell visibility of any observer is a subset of the cell-to-cell visibility for the cell containing the observer





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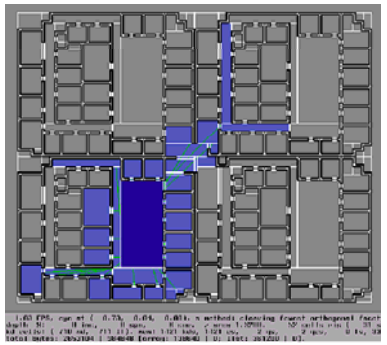
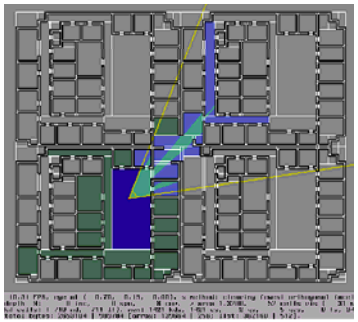
Results



(a) A source cell (dark blue), its cell-to-cell visibility (light blue), and stabbing lines (green).



(d) The same observer, with a 60° view cone. The eye-to-cell visibility is shown in blue; the exact visible area is shown in blue-green. The green cells have been dynamically culled.

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