Photon Mapping

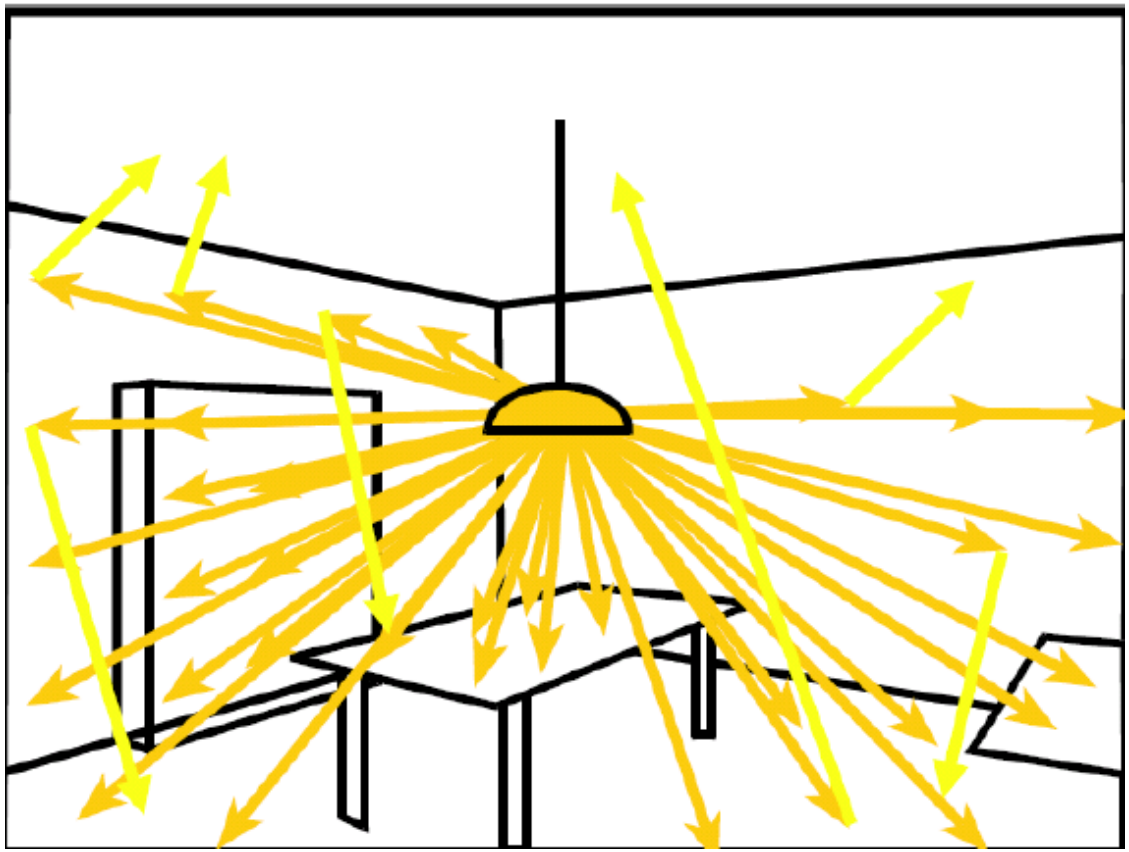
Kadi Bouatouch
IRISA
Email: kadi@irisa.fr

IFSIC INSTITUT DE FORMATION SUPERIEURE en INFORMATIQUE et COMMUNICATION

IRISA

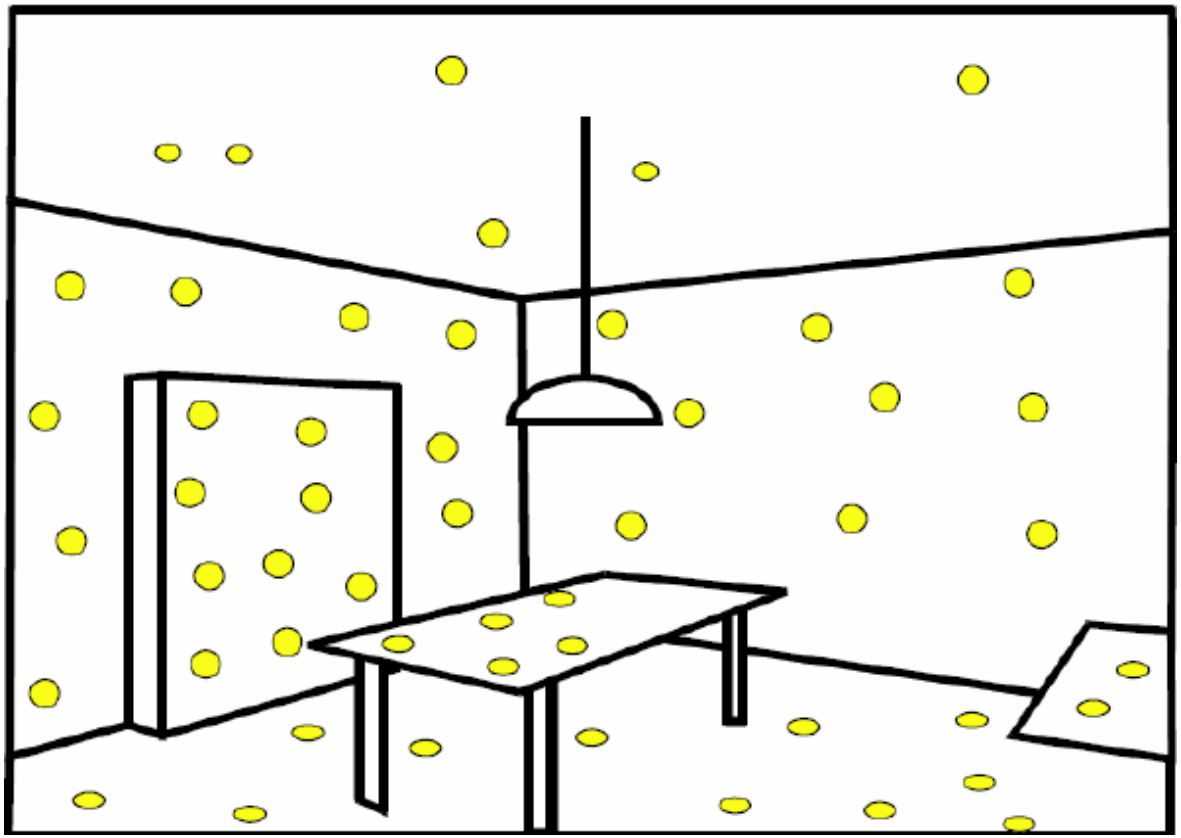
Photon Mapping

Photon emission and transport



Photon Mapping

- Photon caching

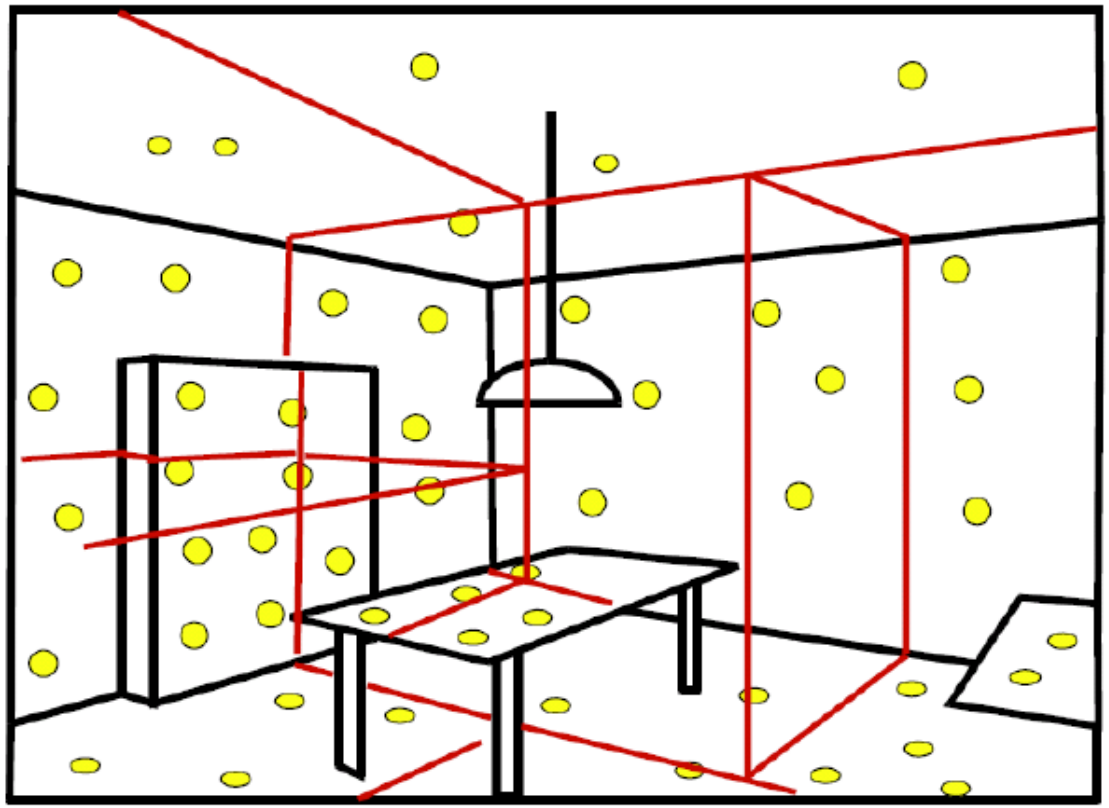


IFSIC INSTITUT DE FORMATION SUPERIEURE en INFORMATIQUE et COMMUNICATION

IRISA

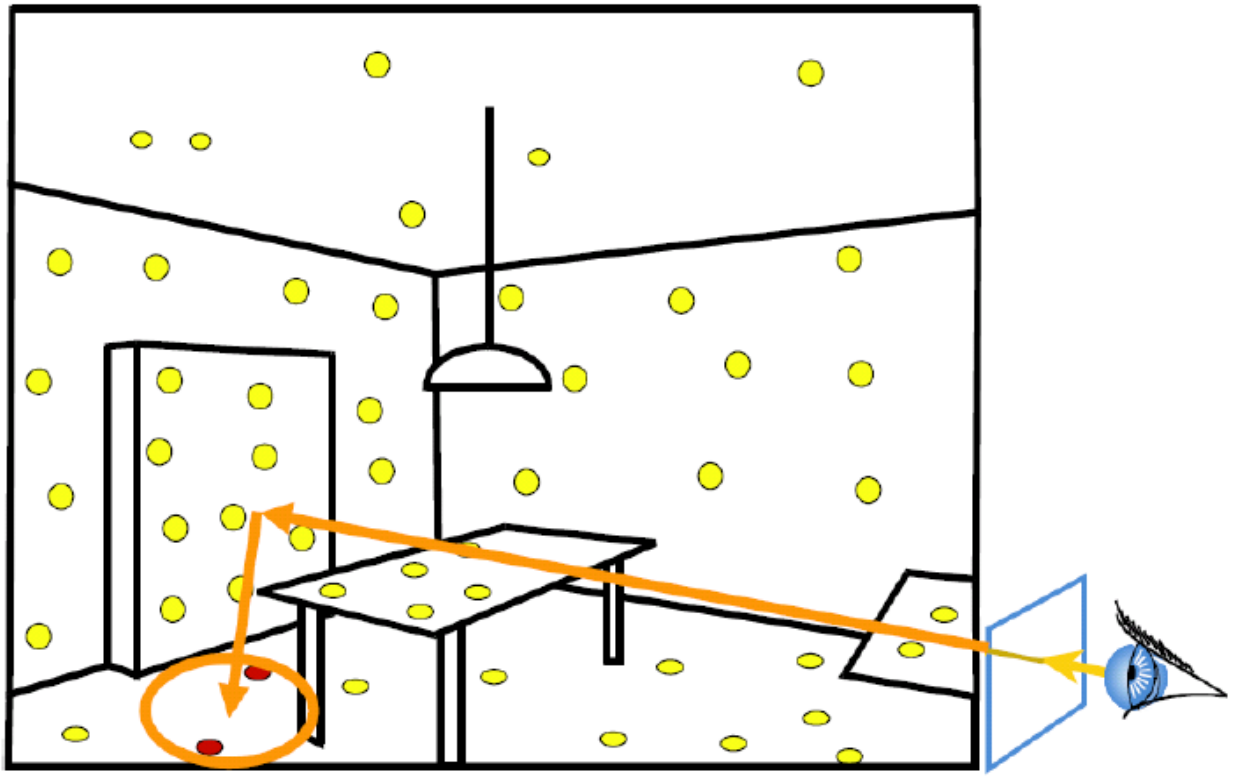
Photon Mapping

- Spatial data structure for fast access



Photon Mapping

- Radiance estimation



Photon Mapping

- Kd-tree
  - Balanced Binary Tree
  - When a splitting dimension of a set is selected, the median of the points in that dimension is chosen as the root node of the tree representing the set
  - The left and right subtrees are constructed from the two sets separated by the median point.
  - The choice of a splitting dimension is based on the distribution of points within the set.

Photon Mapping

- Kd-tree
  - Splitting plane: based upon maximum distance since it can be computed very efficiently.
  - The splitting dimension is thus chosen as the one which has the largest maximum distance between the points.
  - To speed up the balancing process it is convenient to use an array of pointers to the photons.
  - Store the Kd-tree in a linear array.

Photon Mapping

- Kd-tree construction

```
kdtree *balance( points ) {
    Find the cube surrounding the points
    Select dimension dim in which the cube is largest
    Find median of the points in dim
    s1 = all points below median
    s2 = all points above median
    node = median
    node.left = balance( s1 )
    node.right = balance( s2 )
    return node
}
```

IFSIC INSTITUT DE FORMATION SUPERIEURE en INFORMATIQUE et COMMUNICATION

IRISA

Photon Mapping

- Kd-tree: locating the nearest photons
  - Efficiently locating the nearest photons is critical for good performance of the photon map algorithm.
  - Fortunately the simplicity of the kd-tree permits us to implement a simple but quite efficient search algorithm.
  - A generic nearest neighbors search algorithm begins at the root of the kd-tree, and adds photons to a list if they are within a certain distance.
  - For the n nearest neighbors the list is sorted such that the photon that is furthest away can be deleted if the list contains n photons and a new closer photon is found.

Photon Mapping

- Kd-tree: locating the nearest photons
  - Instead of naive sorting of the full list it is better to use a max-heap.
  - A max-heap (also known as a priority queue) is a very efficient way of keeping track of the element that is furthest away from the point of interest. This furthest element is placed as the first position of the heap.
  - When the max-heap is full, we can use the distance d to the root element (ie. the photon that is furthest away, say the first element of the heap) to adjust the range of the query.
  - Thus we skip parts of the kd-tree that are further away than d.

Photon Mapping

Given the photon map, a position x and a max search distance d2 this recursive function returns a heap h with the N nearest photons.
Call with locate_photons(1) to initiate search at the root of the kd-tree

```
locate_photons( p ) {
    if ( 2p + 1 < number of photons ) {
        examine child nodes
            Compute distance to plane (just a subtract)
      λ = signed distance to splitting plane of node n
      if (λ < 0) {
          We are left of the plane - search left subtree first
        locate_photons( 2p )
        if (λ² < d² )
          locate_photons( 2p + 1 )     check right subtree
      } else {
          We are right of the plane - search right subtree first
        locate_photons( 2p + 1 )
        if (λ² < d² )
          locate_photons( 2p )     check left subtree
        }
    }
```

IFSIC INSTITUT DE FORMATION SUPERIEURE en INFORMATIQUE et COMMUNICATION

IRISA

Photon Mapping

```
                    Compute true squared distance to photon
        λ² = squared distance from photon p to x
        if (λ² < d² ) {          Check if the photon is close enough?
            insert photon into max heap h
                                Adjust maximum distance to prune the search
            if (number of photons in the max heap > N) {
                                save the new maximum distance
              d² = squared distance to photon in root node of h
            }
        }
}
```

Photon Mapping

- Reflection, Transmission or Absorption: Phong

$$f_r(\Theta_i \leftrightarrow \Theta_r) = k_d + k_s \cos^n(\Theta_r, \Theta_s)$$

or also:

$$f_r(\Theta_i \leftrightarrow \Theta_r) = \frac{\rho_d}{\pi} + \frac{\rho_s(n+2)}{2\pi}\cos^n(\Theta_r, \Theta_s) \text{ with } \rho_d = \pi k_d \text{ and } \rho_s = \frac{2\pi}{n+2}k_s$$

Energy conservation: $\rho_d + \rho_s \leq 1$. ρ_d and ρ_s are the maximum energy ($\Theta_i = n_x$) reflected through the diffuse part and specular part respectively. In other words:

$$\rho_d = \int_{\Omega_r} k_d \cos(n_x, \Psi) d\omega_\Psi \qquad \rho_s = \int_{\Omega_r} k_s \cos^{n+1}(n_x, \Psi) d\omega_\Psi \text{ (to compute, see 30)}$$

Photon Mapping

The probabilities for specular and diffuse reflection can be based on the total energy reflected by each type of reflection or on the maximum energy reflected in any color band. If we base the decision on maximum energy, we can for example compute the probability P_d for diffuse reflection as

$$P_d = \frac{\max(d_r P_r, d_g P_g, d_b P_b)}{\max(P_r, P_g, P_b)}$$

where (d_r, d_g, d_b) are the diffuse reflection coefficients in the red, green, and blue color bands, and (P_r, P_g, P_b) are the powers of the incident photon in the same three color bands.

Similarly, the probability P_s for specular reflection is

$$P_s = \frac{\max(s_r P_r, s_g P_g, s_b P_b)}{\max(P_r, P_g, P_b)}$$

where (s_r, s_g, s_b) are the specular reflection coefficients.

Photon Mapping

The probability of absorbtion is $P_a = 1 - P_d - P_s$. With these probabilities, the decision of which type of reflection or absorption should be chosen takes the following form:

$$\begin{aligned} \xi \in [0, P_d] &\longrightarrow \text{diffuse reflection} \\ \xi \in]P_d, P_s + P_d] &\longrightarrow \text{specular reflection} \\ \xi \in]P_s + P_d, 1] &\longrightarrow \text{absorption} \end{aligned}$$

The power of the reflected photon needs to be adjusted to account for the probability of survival. If, for example, specular reflection was chosen in the example above, the power P_{refl} of the reflected photon is:

$$\begin{aligned} P_{refl,r} &= P_{inc,r}\, s_r / P_s \\ P_{refl,g} &= P_{inc,g}\, s_g / P_s \\ P_{refl,b} &= P_{inc,b}\, s_b / P_s \end{aligned}$$

where P_{inc} is the power of the incident photon.

The computed probabilities again ensure us that we do not waste time emitting photons with very low power.

Photon Mapping

- **The illumination model**
  - The BRDF is separated into a sum of two components: A specular/glossy, $f_{r,s}$, and a diffuse, $f_{r,d}$
  - The incoming radiance is classified using three components:
    - $L_{i,l}()$ is direct illumination by light coming from the light sources.
    - $L_{i,c}()$ is caustics: indirect illumination from the light sources via specular reflection or transmission
    - $L_{i,d}()$ is indirect illumination from the light sources which has been reflected diffusely at least once.

Photon Mapping

- **The illumination model**

The incoming radiance is the sum of these three components:

$$L_i(x,\omega') = L_{i,l}(x,\omega') + L_{i,c}(x,\omega') + L_{i,d}(x,\omega')\,. \tag{4.18}$$

By using the classifications of the BRDF and the incoming radiance we can split the expression for reflected radiance into a sum of four integrals:

$$\begin{aligned}
L_r(x,\omega) &= \int_{\Omega_x} f_r(x,\omega',\omega) L_i(x,\omega') \cos\theta_i \, d\omega_i' \\
&= \int_{\Omega_x} f_r(x,\omega',\omega) L_{i,l}(x,\omega') \cos\theta_i \, d\omega_i' + \\
&\quad \int_{\Omega_x} f_{r,s}(x,\omega',\omega)(L_{i,c}(x,\omega') + L_{i,d}(x,\omega')) \cos\theta_i \, d\omega_i' + \\
&\quad \int_{\Omega_x} f_{r,d}(x,\omega',\omega) L_{i,c}(x,\omega') \cos\theta_i \, d\omega_i' + \\
&\quad \int_{\Omega_x} f_{r,d}(x,\omega',\omega) L_{i,d}(x,\omega') \cos\theta_i \, d\omega_i'\,.
\end{aligned} \tag{4.19}$$

Photon Mapping

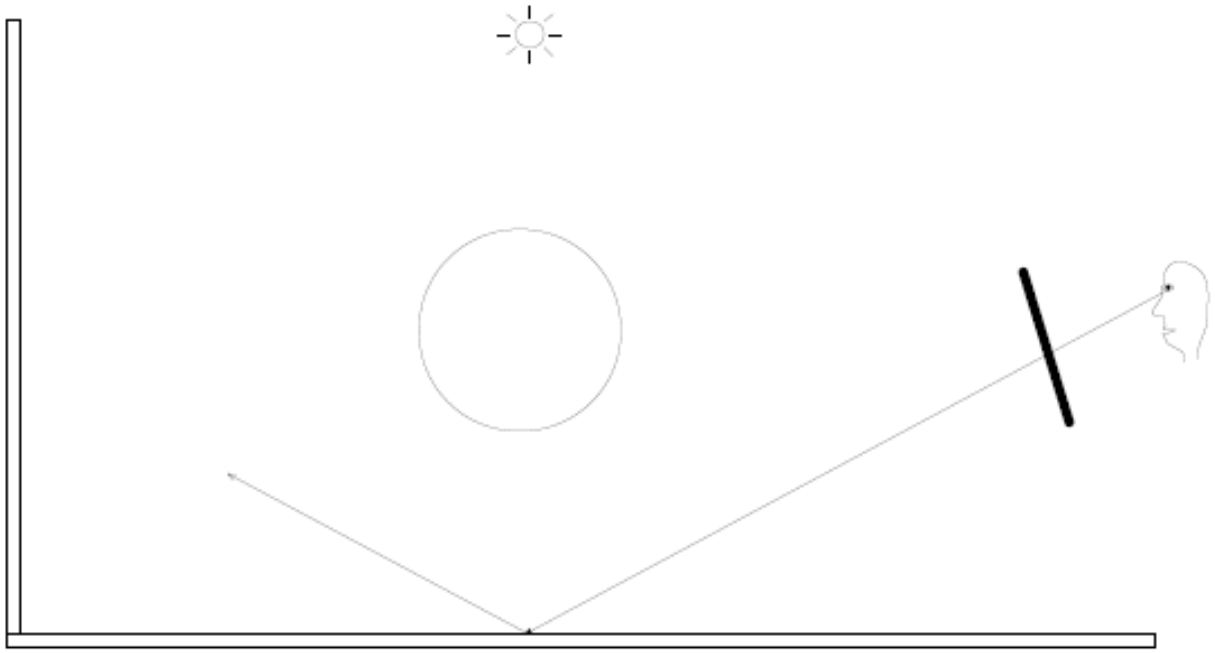
- **Specular and Glossy Reflection**

$$\int_{\Omega_x} f_{r,s}(x,\omega',\omega)(L_{i,c}(x,\omega') + L_{i,d}(x,\omega'))\cos\theta_i \, d\omega'_i$$

  - The photon map is not used in the evaluation of this integral since it is strongly dominated by $f_{r,s}$ which has a narrow peak around the mirror direction.
  - Using the photon map to optimize the integral would require a huge number of photons in order to make a useful classification of the different directions within the narrow peak of $f_{r,s}$
  - To save memory this strategy is not used and the integral is evaluated using standard Monte Carlo ray tracing optimized with importance sampling based on $f_{r,s}$.
  - This is still quite efficient for glossy surfaces and the integral can in most situations be computed using only a small number of sample rays.

Photon Mapping

- **Specular and Glossy Reflection**

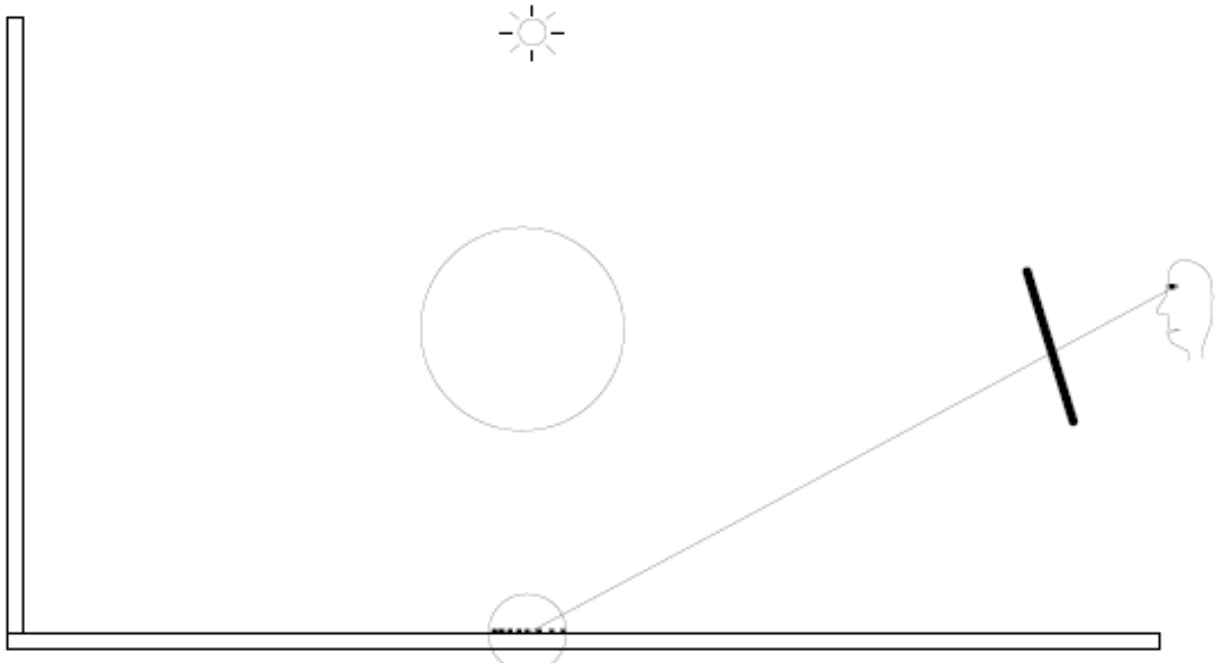


Photon Mapping

- **Caustics** $\int_{\Omega_x} f_{r,d}(x, \omega', \omega) L_{i,c}(x, \omega') \cos\theta_i \, d\omega'_i \, .$
  - Evaluation: dependent on whether an accurate or an approximate computation is required.
  - Accurate computation: the term is solved by using a radiance estimate from the caustics photon map.
  - The number of photons in the caustics photon map is high and we can expect good quality of the estimate.
  - Caustics are never computed using Monte Carlo ray tracing since this is a very inefficient method when it comes to rendering caustics.
  - The approximate evaluation of the integral is included in the radiance estimate from the global photon map.

Photon Mapping

- **Caustics**



Photon Mapping

- Multiple Diffuse Reflection
  - This term represents incoming light that has been reflected diffusely at least once since it left the light source. The light is then reflected diffusely by the surface (using $f_{r,d}$).
  - Consequently the resulting illumination is very “soft”.
  - The approximate evaluation of this integral is a part of the radiance estimate based on the global photon map.
  - The accurate evaluation of the integral is calculated using Monte Carlo ray tracing optimized using the BRDF

Photon Mapping

- Multiple Diffuse Reflection