Photons

Radiance Estimate

$$L(x,\vec{\omega}) \;=\; \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega})L'(x,\vec{\omega}')\cos\theta'\,d\omega$$

Radiance Estimate

$$
\begin{aligned}
L(x,\vec{\omega}) &= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta' \, d\omega \\
&= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{d\omega \ \cos\theta' dA} \cos\theta' d\omega
\end{aligned}
$$

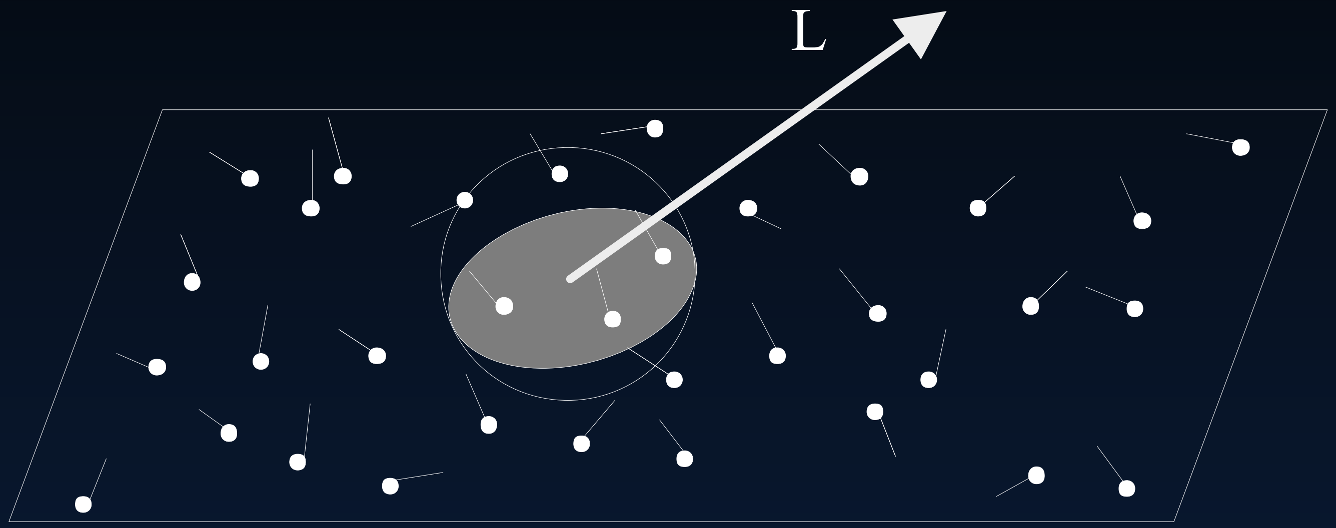
Radiance Estimate

$$
\begin{aligned}
L(x,\vec{\omega}) &= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta' \, d\omega \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{d\omega \; \cos\theta' dA} \; \cos\theta' d\omega \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{dA}
\end{aligned}
$$

Radiance Estimate

$$
\begin{aligned}
L(x,\vec{\omega}) &= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta' \, d\omega \\
&= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{d\omega \ \cos\theta' dA} \ \cos\theta' d\omega \\
&= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{dA} \\
&\approx \sum_{p=1}^{n} f_r(x,\vec{\omega}'_p,\vec{\omega}) \frac{\Delta\Phi_p(x,\vec{\omega}'_p)}{\pi r^2}
\end{aligned}
$$

Radiance Estimate

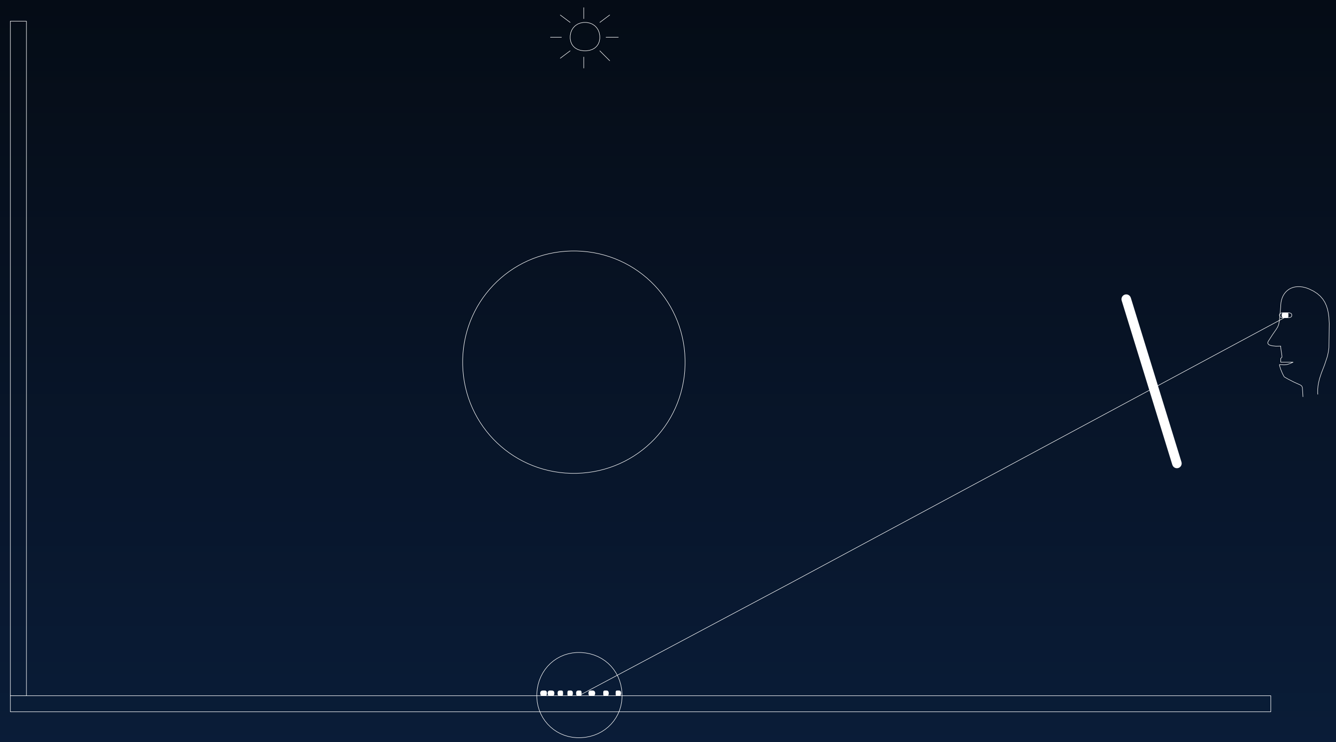


The photon map datastructure

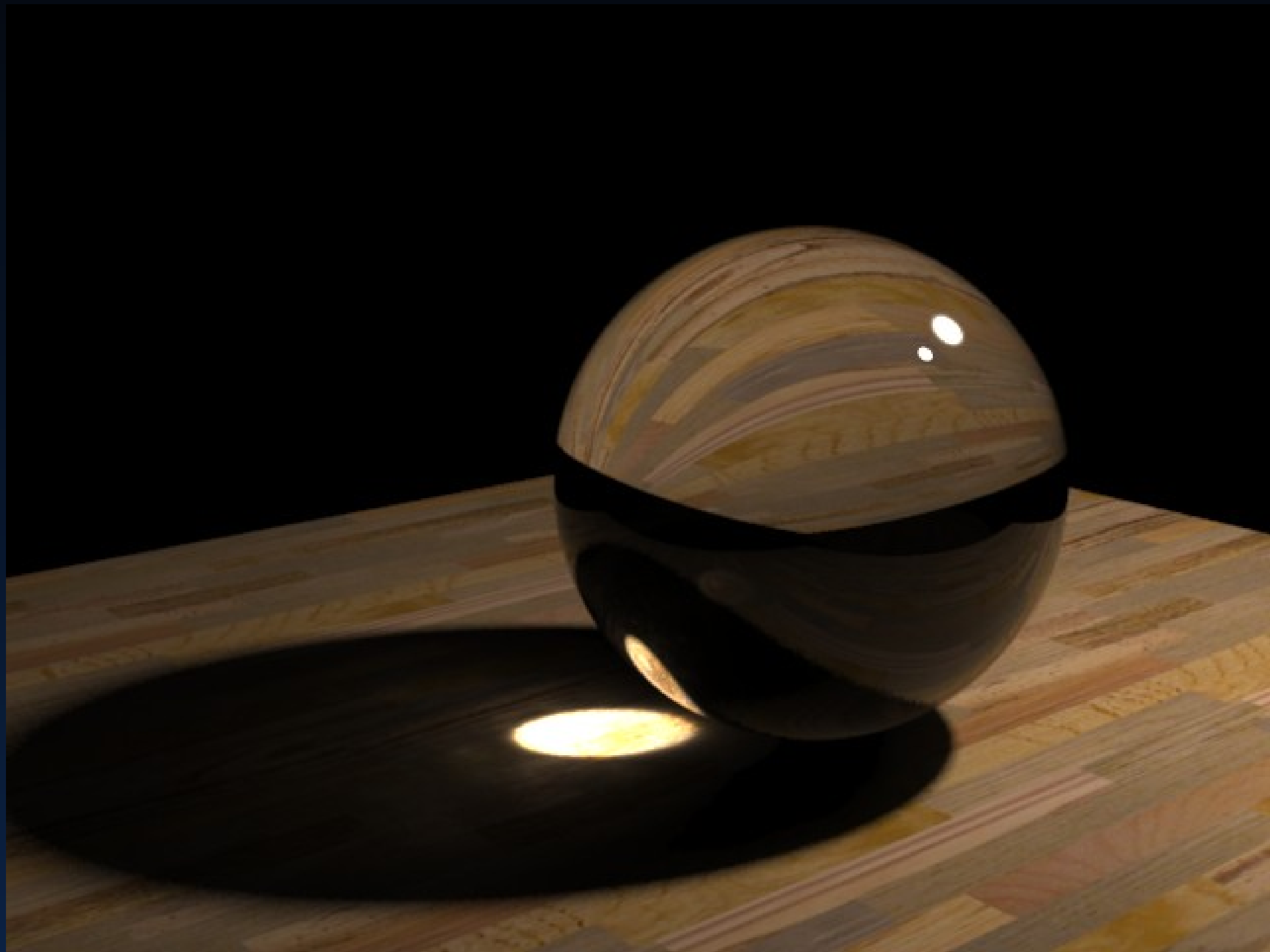
The photons are stored in a left balanced kd-tree

```
struct photon = {
  float position[3];
  rgbe power;           // power packed as 4 bytes
  char phi, theta;      // incoming direction
  short flags;
}
```

Rendering: Caustics

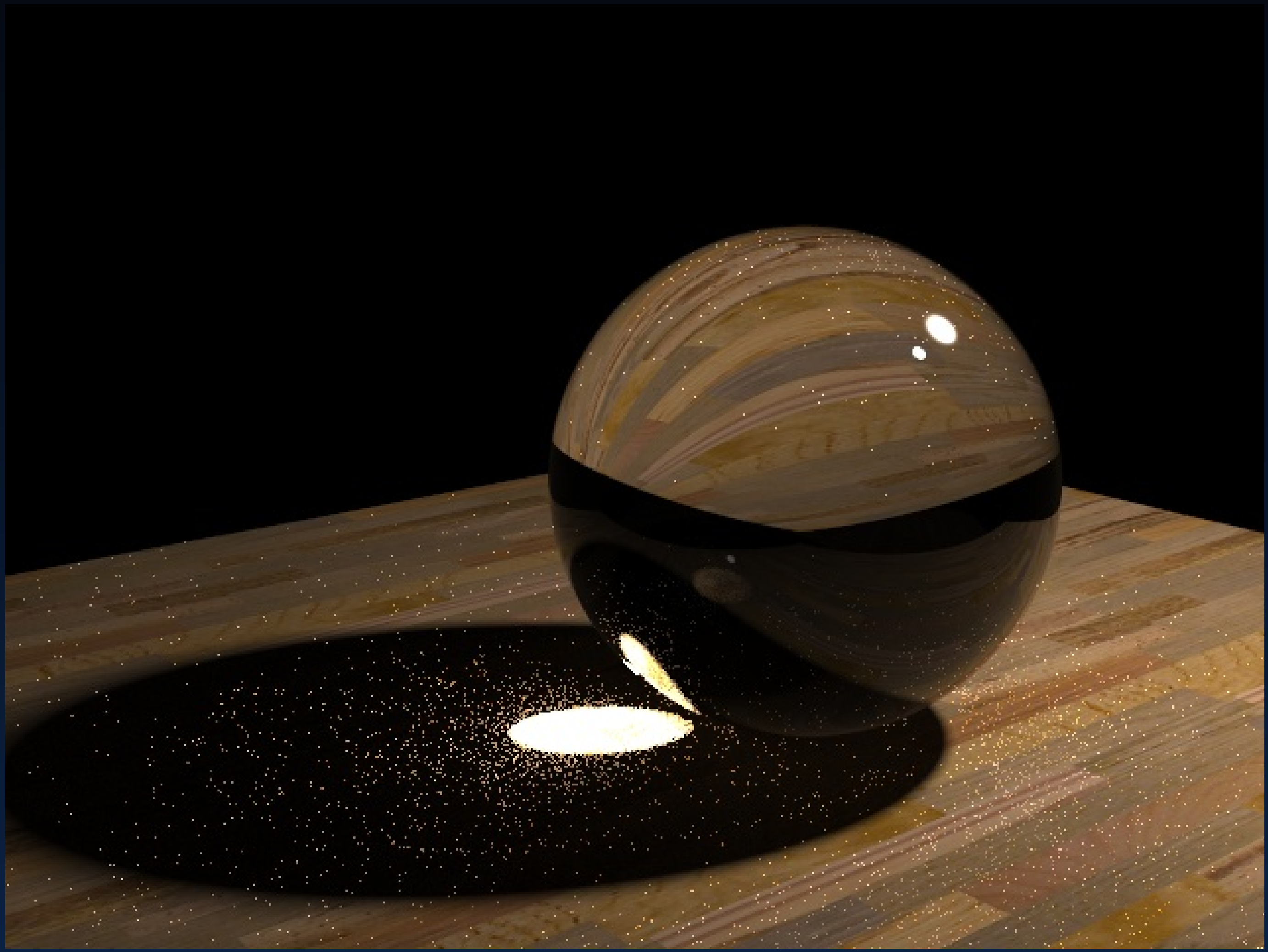


Caustic from a Glass Sphere



Photon Mapping: 10000 photons / 50 photons in radiance estimate

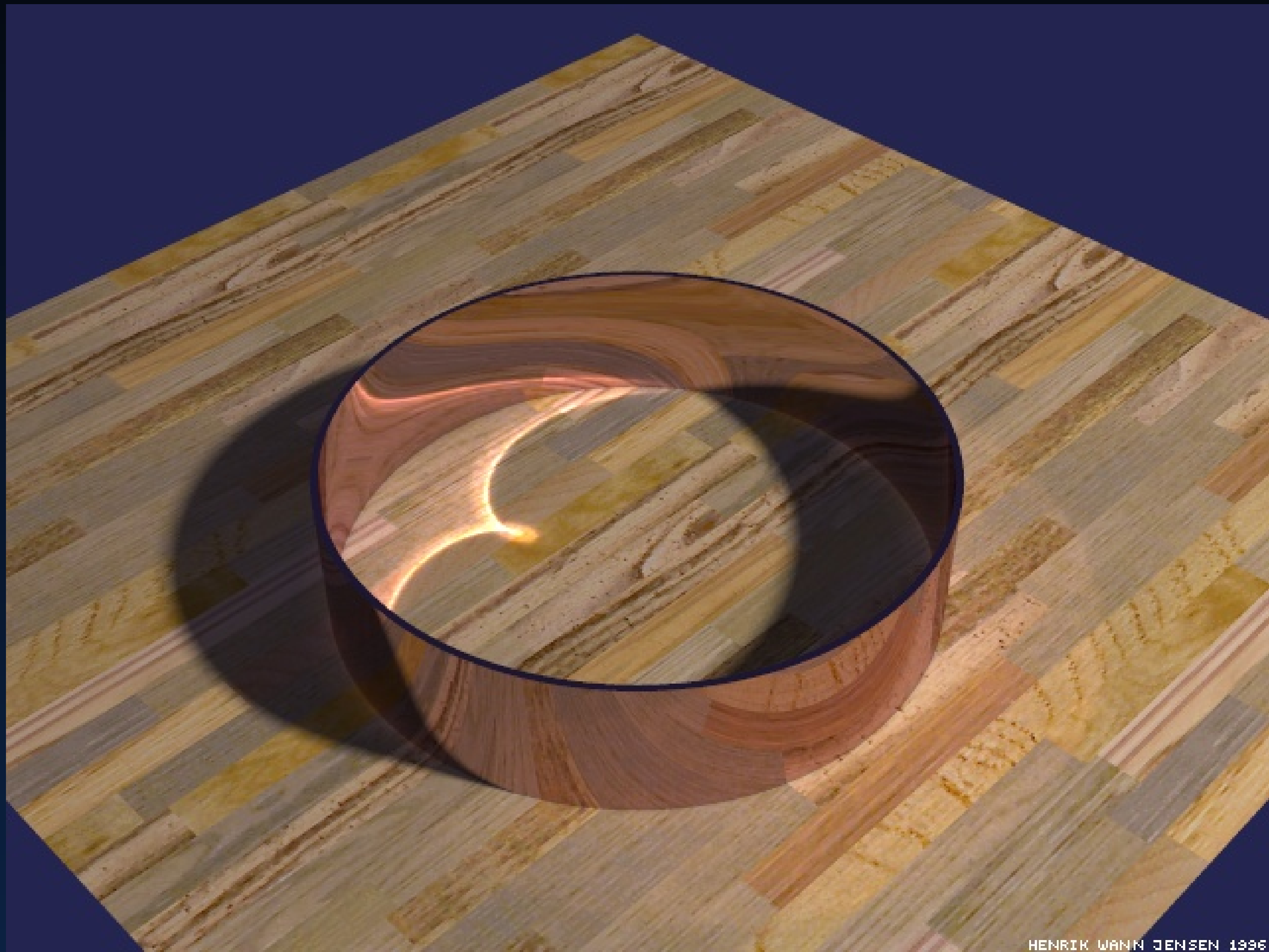
Caustic from a Glass Sphere



Path Tracing: 1000 paths/pixel

Sphereflake Caustic

Reflection Inside A Metal Ring

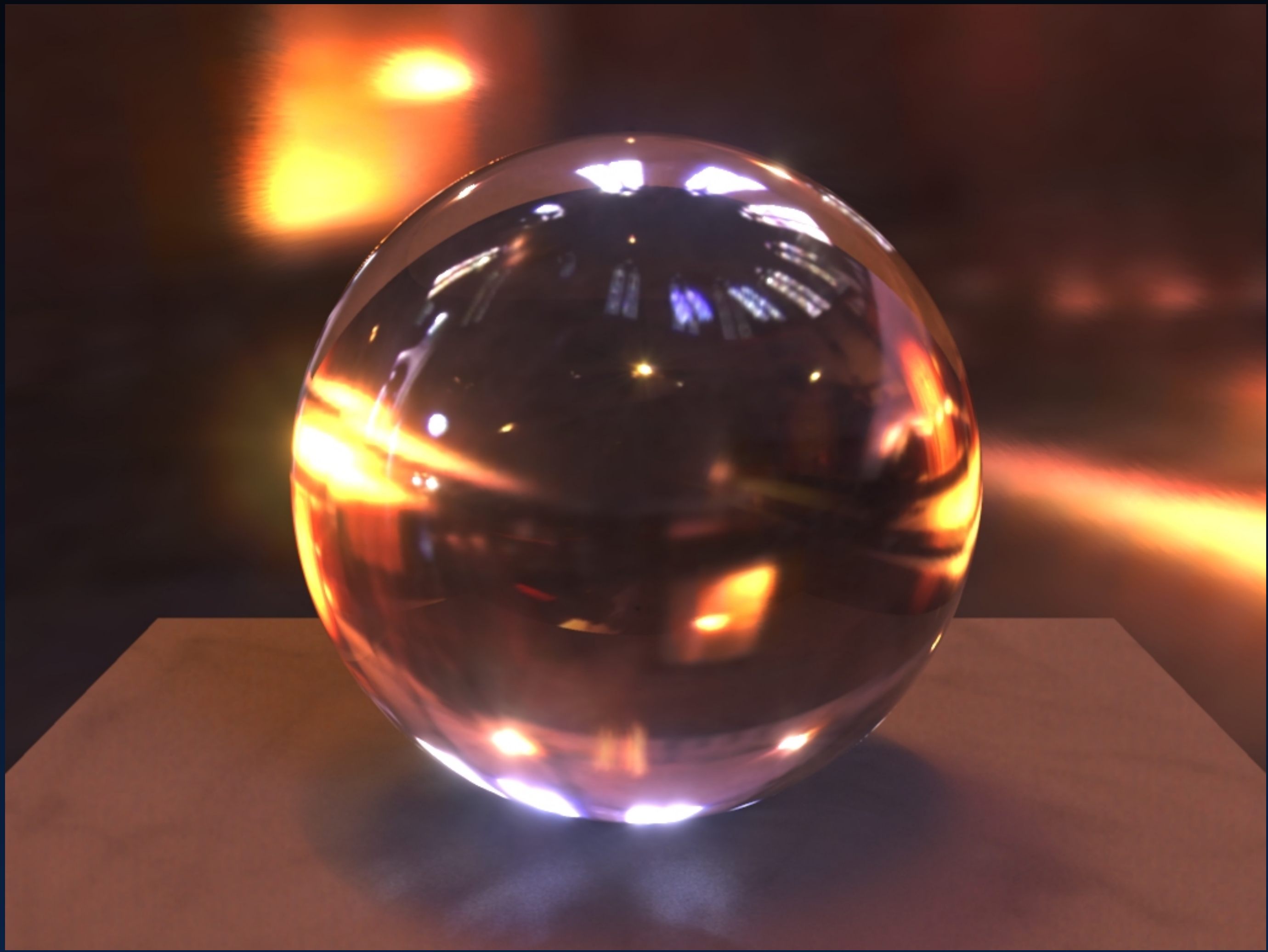


50000 photons / 50 photons in radiance estimate

Caustics On Glossy Surfaces

340000 photons / $\approx$ 100 photons in radiance estimate

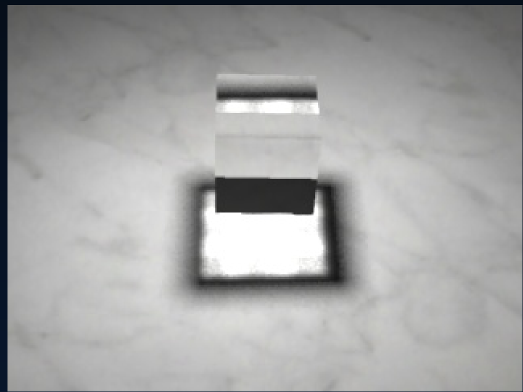
HDR environment illumination



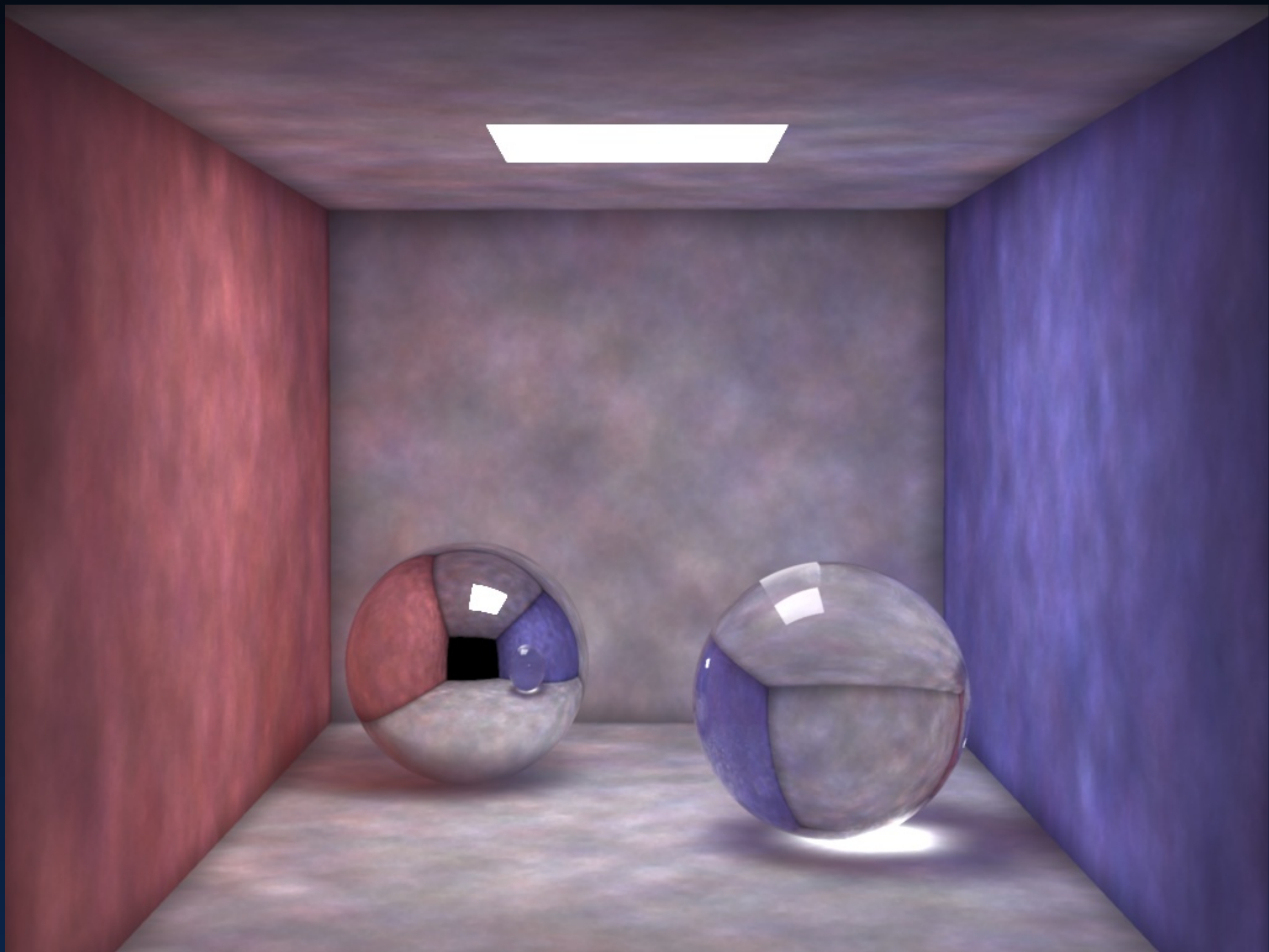
Using lightprobe from www.debevec.org

Cognac Glass

Cube Caustic

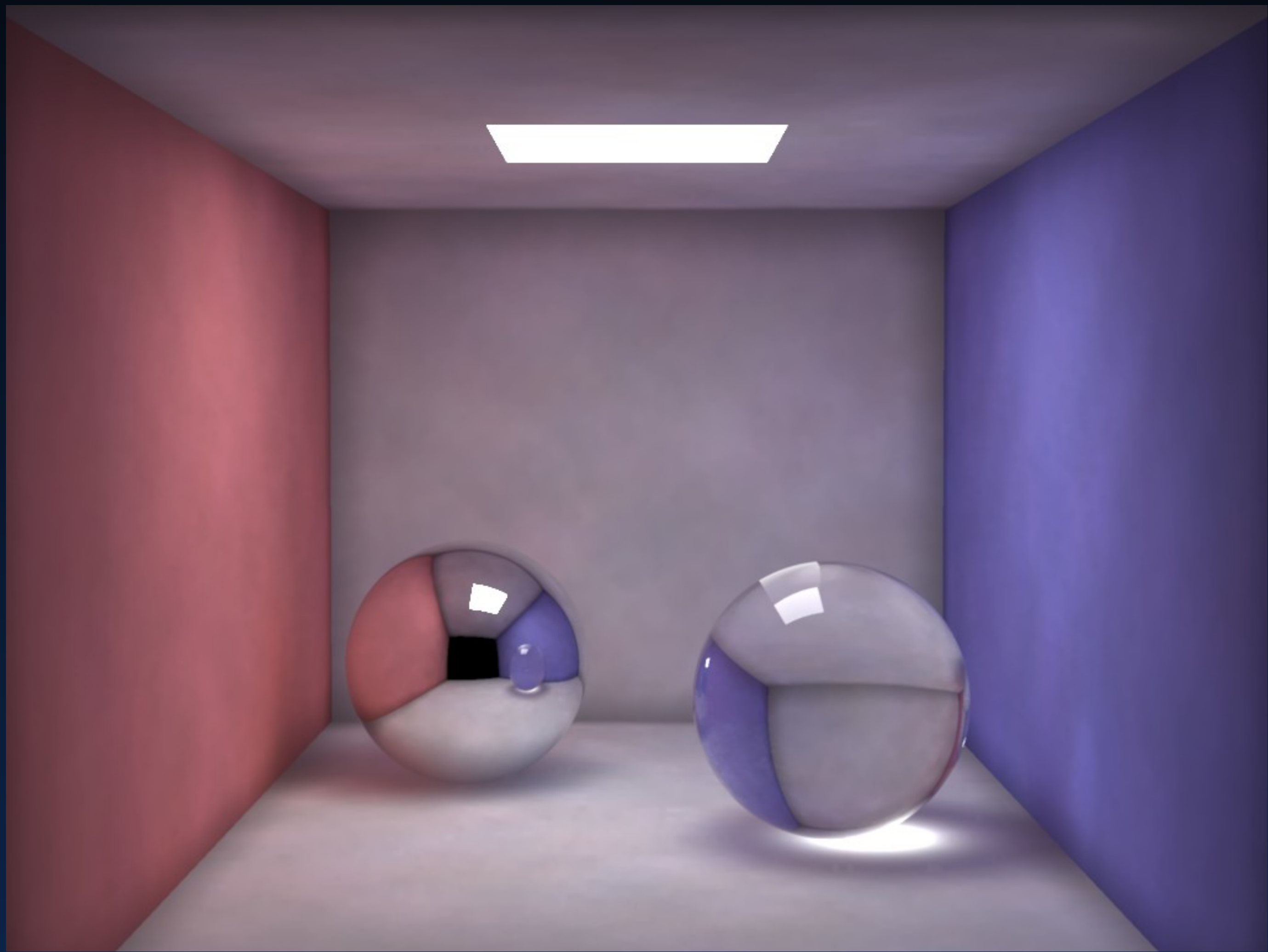


Global Illumination



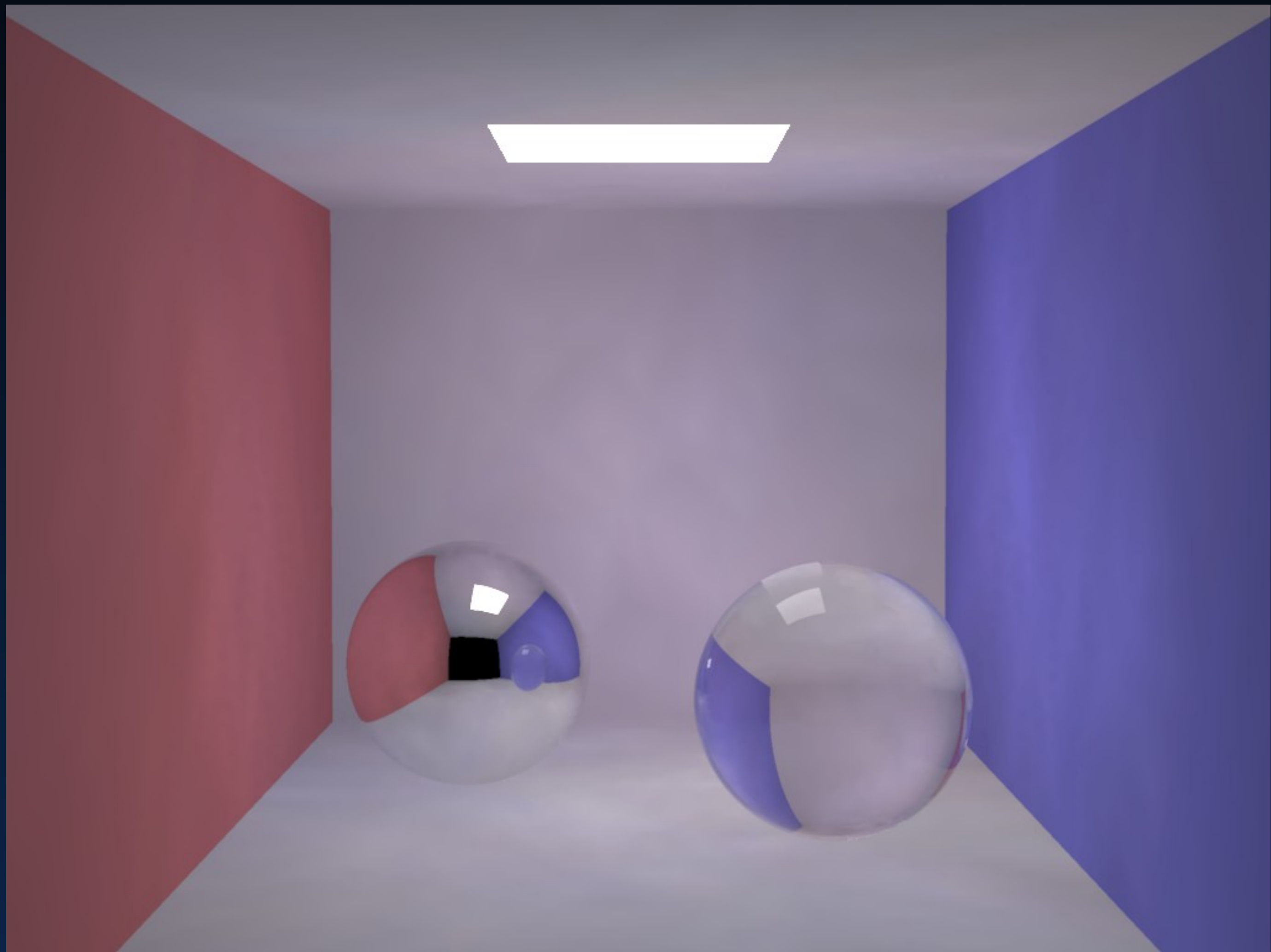
100000 photons / 50 photons in radiance estimate

Global Illumination



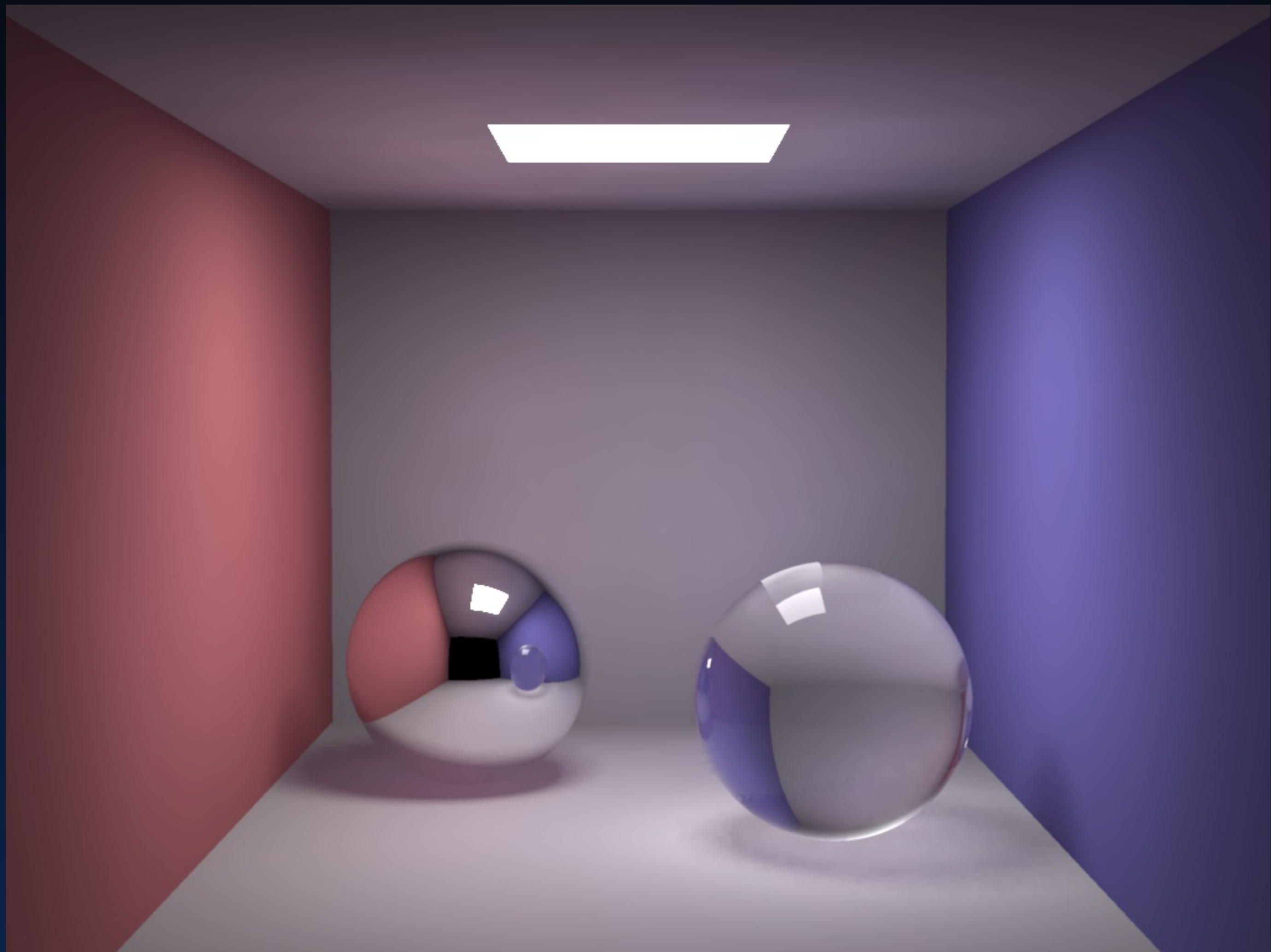
500000 photons / 500 photons in radiance estimate

Fast estimate



200 photons / 50 photons in radiance estimate

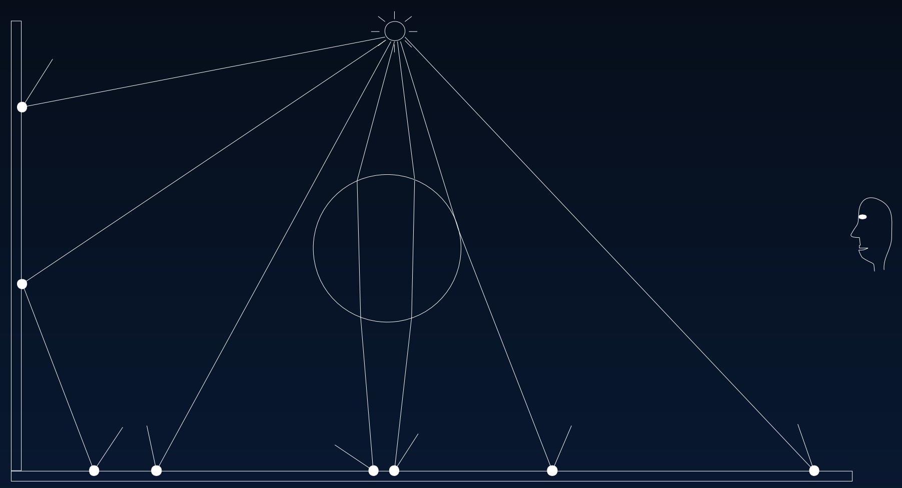
Indirect illumination



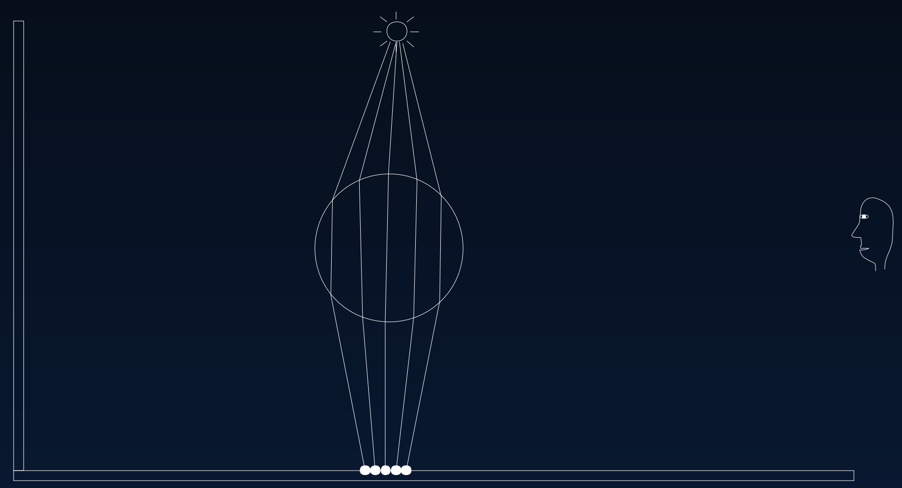
10000 photons / 500 photons in radiance estimate

Global Illumination

Global Illumination



global photon map



caustics photon map

Photon tracing

- Photon emission
- Photon scattering
- Photon storing

Photon emission

Given Φ Watt lightbulb.
Emit N photons.
Each photon has the power $\frac{\Phi}{N}$ Watt.

- Photon power depends on the number of emitted photons. Not on the number of photons in the photon map.

What is a photon?

- Flux (power) - not radiance!
- Collection of physical photons
  - A fraction of the light source power
  - Several wavelengths combined into one entity

Diffuse point light

Generate random direction
Emit photon in that direction

```
// Find random direction
do {
  x = 2.0*random()-1.0;
  y = 2.0*random()-1.0;
  z = 2.0*random()-1.0;
} while ( (x*x + y*y + z*z) > 1.0 );
```

Example: Diffuse square light

- Generate random position p on square
- Generate diffuse direction d
- Emit photon from p in direction d

```
// Generate diffuse direction
u = random();
v = 2*π*random();
d = vector( cos(v)√u, sin(v)√u, √(1-u) );
```

$$u = \mathtt{random();}$$
$$v = \mathtt{2{*}}\pi\mathtt{{*}random();}$$
$$d = \mathtt{vector(}\ cos(v)\sqrt{u},\ sin(v)\sqrt{u},\ \sqrt{1-u}\ \mathtt{);}$$

Surface interactions

The photon is

- Stored (at diffuse surfaces) and
- Absorbed (A) or
- Reflected (R) or
- Transmitted (T)

$$A + R + T = 1.0$$

Photon scattering

The simple way:

Given incoming photon with power Φ_p

Reflect photon with the power $R * \Phi_p$

Transmit photon with the power $T * \Phi_p$

Photon scattering

The simple way:

Given incoming photon with power Φ_p

Reflect photon with the power $R * \Phi_p$

Transmit photon with the power $T * \Phi_p$

- Risk: Too many low-powered photons - wasteful!
- When do we stop (systematic bias)?
- Photons with similar power is a good thing.

Russian Roulette

- Statistical technique
- Known from Monte Carlo particle physics
- Introduced to graphics by Arvo and Kirk in 1990

Russian Roulette

Probability of termination: p

Russian Roulette

Probability of termination: p

$$E\{X\}$$

Russian Roulette

Probability of termination: p

$$E\{X\} = p \cdot 0$$

Russian Roulette

Probability of termination: p

$$E\{X\} = p \cdot 0 + (1 - p)$$

Russian Roulette

Probability of termination: p

$$E\{X\} = p \cdot 0 + (1-p) \cdot \frac{E\{X\}}{1-p}$$

Russian Roulette

Probability of termination: p

$$E\{X\} = p \cdot 0 + (1-p) \cdot \frac{E\{X\}}{1-p} = E\{X\}$$

Russian Roulette

Probability of termination: p

$$E\{X\} = p \cdot 0 + (1-p) \cdot \frac{E\{X\}}{1-p} = E\{X\}$$

Terminate un-important photons and still get the correct result.

Russian Roulette Example

Surface reflectance: R = 0.5
Incoming photon: Φ_p = 2 W

```
r = random();
if ( r < 0.5 )
  reflect photon with power 2 W
else
  photon is absorbed
```

Russian Roulette Intuition

Surface reflectance: R = 0.5
200 incoming photons with power: Φ_p = 2 Watt

Reflect 100 photons with power 2 Watt instead of 200 photons with power 1 Watt.

Russian Roulette

- Very important!
- Use to eliminate un-important photons
- Gives photons with similar power :)

Sampling a BRDF

$$f_r(x, \vec{\omega}_i, \vec{\omega}_o) = w_1 f_{r,1}(x, \vec{\omega}_i, \vec{\omega}_o) + w_2 f_{r,2}(x, \vec{\omega}_i, \vec{\omega}_o)$$

Sampling a BRDF

$$f_r(x, \vec{\omega}_i, \vec{\omega}_o) = w_1 \cdot f_{r,d} + w_2 \cdot f_{r,s}$$

r = random()$\cdot(w_1 + w_2)$;

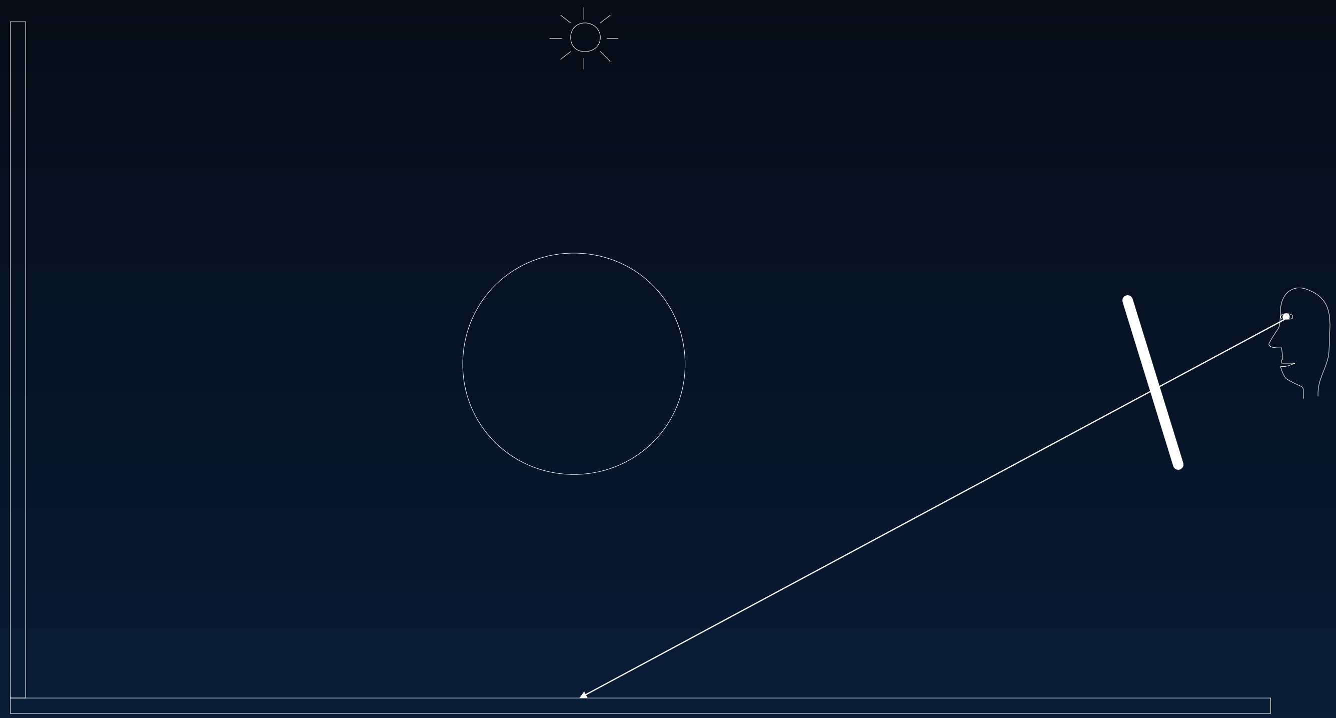
if (r $< w_1$)

  reflect diffuse photon

else

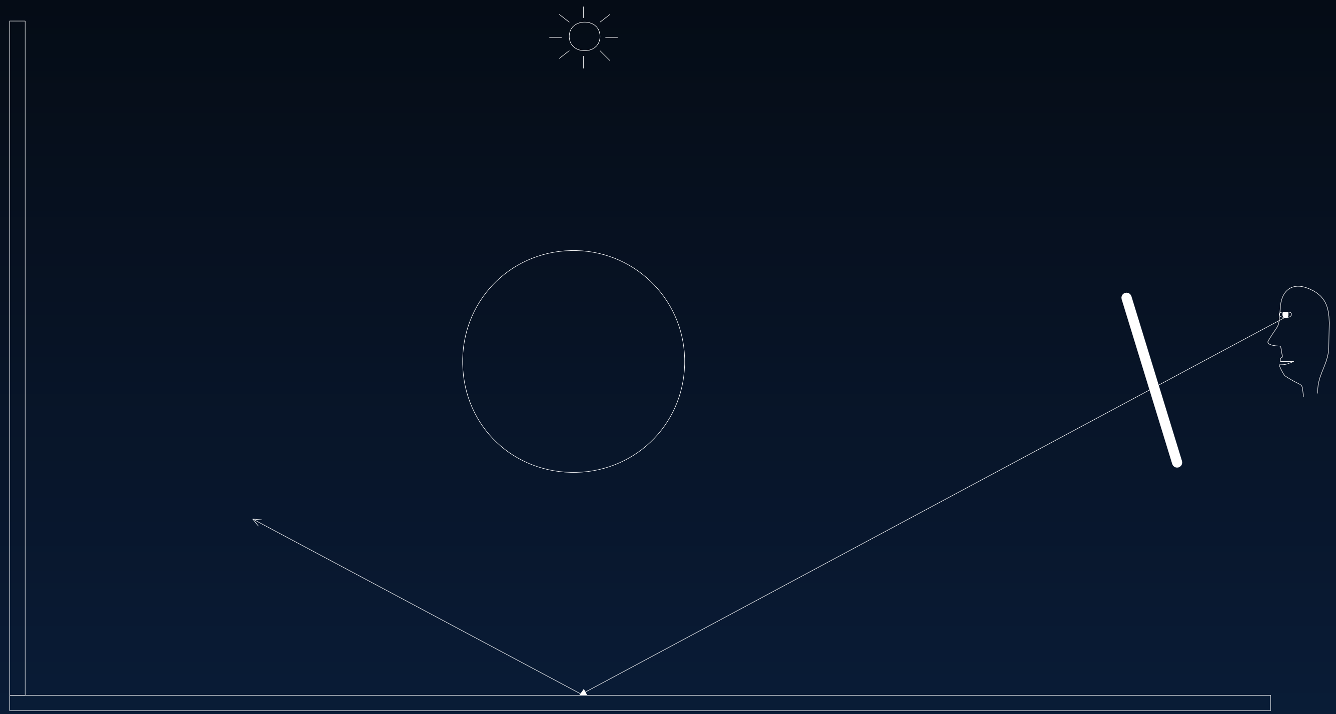
  reflect specular

Rendering

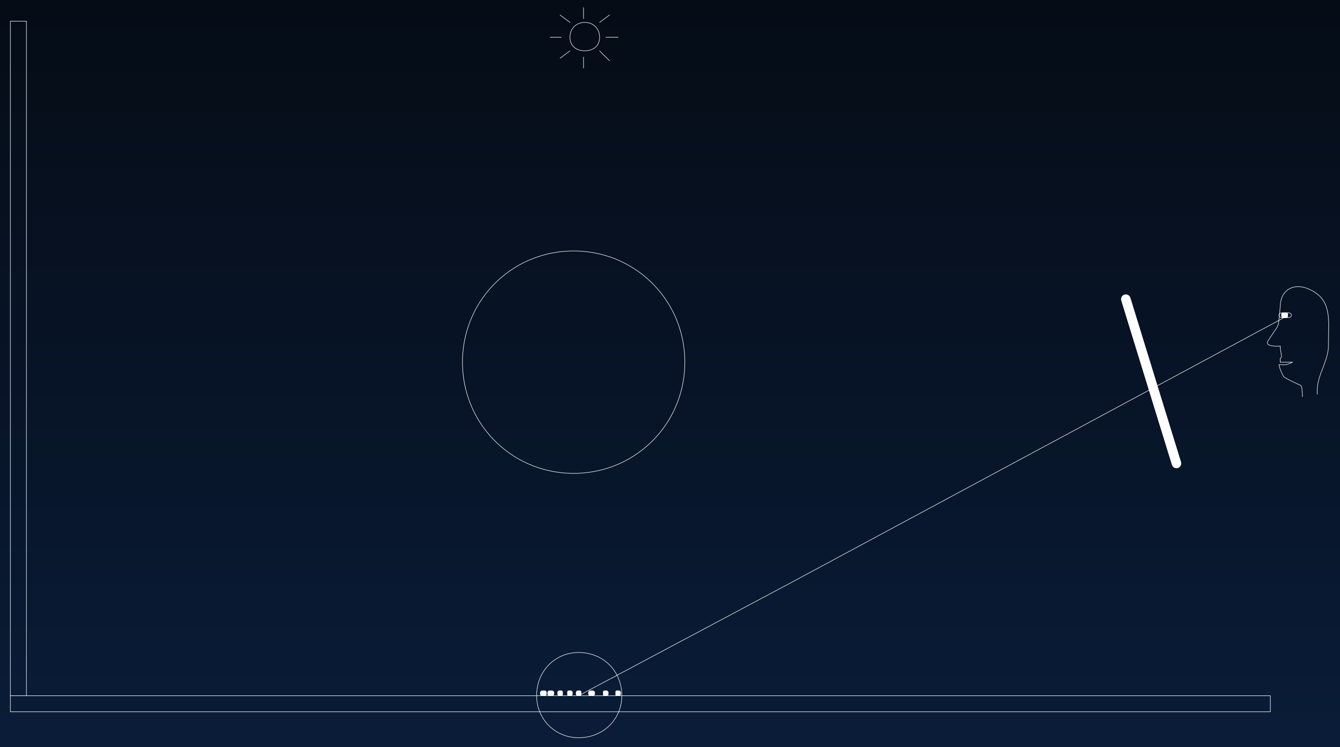


Direct Illumination

Specular Reflection



Caustics



Indirect Illumination

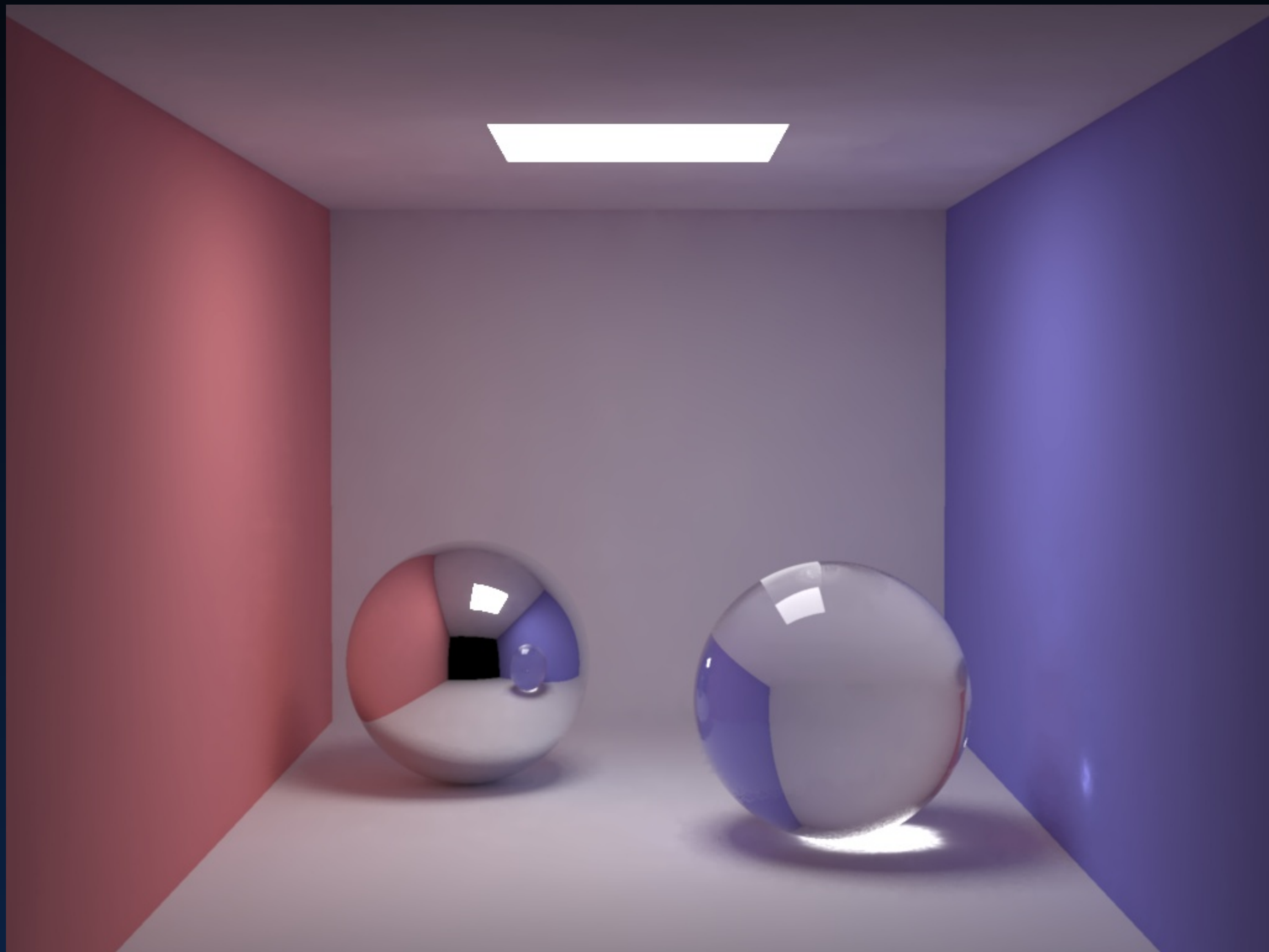
Rendering Equation Solution

$$
\begin{aligned}
L_r(x,\vec{\omega}) &= \int_{\Omega_x} f_r(x,\vec{\omega}',\vec{\omega}) L_i(x,\vec{\omega}') \cos\theta_i \, d\omega_i' \\
&= \int_{\Omega_x} f_r(x,\vec{\omega}',\vec{\omega}) L_{i,l}(x,\vec{\omega}') \cos\theta_i \, d\omega_i' + \\
& \quad \int_{\Omega_x} f_{r,s}(x,\vec{\omega}',\vec{\omega}) (L_{i,c}(x,\vec{\omega}') + L_{i,d}(x,\vec{\omega}')) \cos\theta_i \, d\omega_i' + \\
& \quad \int_{\Omega_x} f_{r,d}(x,\vec{\omega}',\vec{\omega}) L_{i,c}(x,\vec{\omega}') \cos\theta_i \, d\omega_i' + \\
& \quad \int_{\Omega_x} f_{r,d}(x,\vec{\omega}',\vec{\omega}) L_{i,d}(x,\vec{\omega}') \cos\theta_i \, d\omega_i' \,.
\end{aligned}
$$

Features

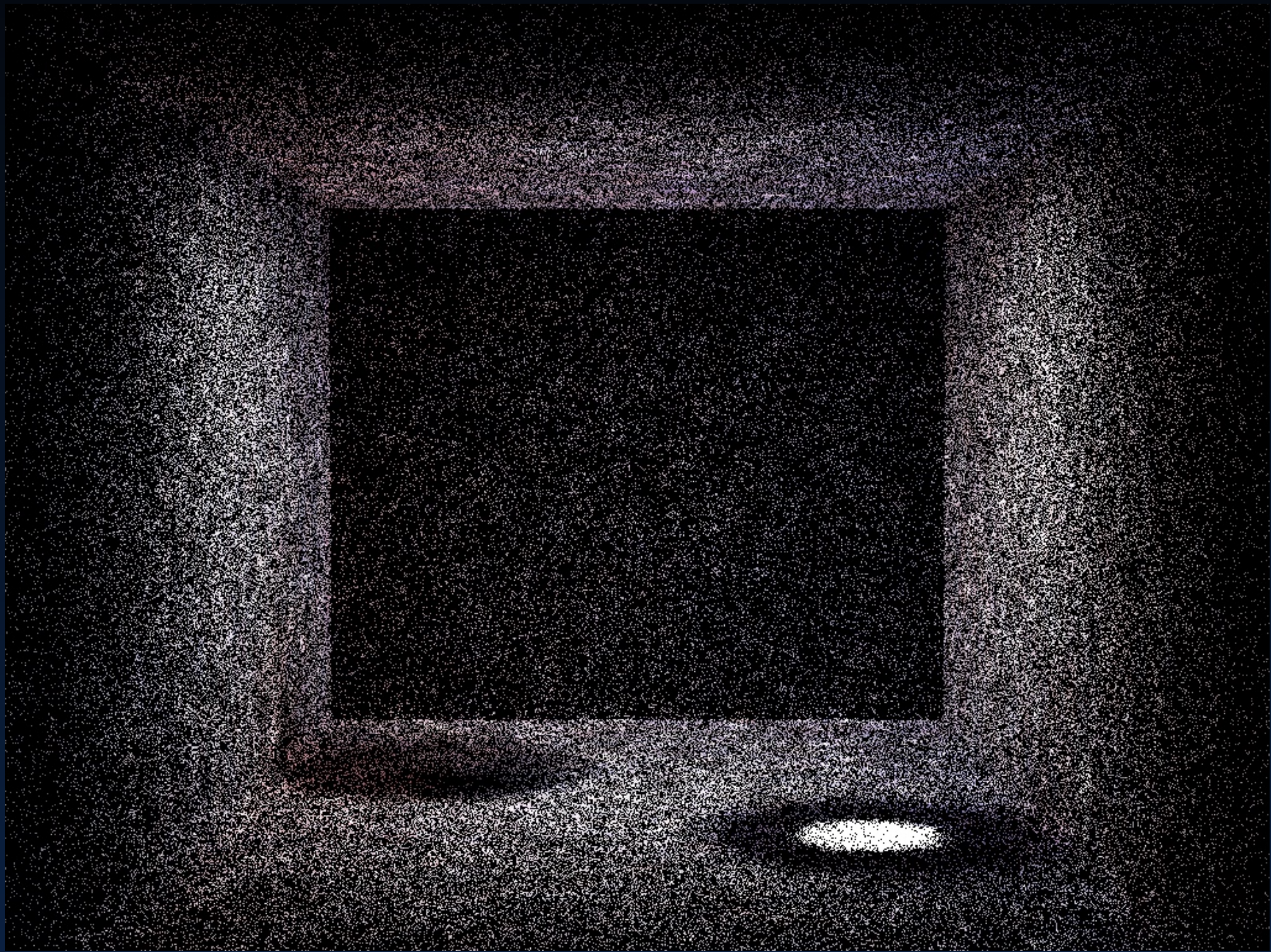
- Photon tracing is unbiased
  - ⋆ Radiance estimate is biased but consistent
  - ⋆ The reconstruction error is local
- Illumination representation is decoupled from the geometry

Box



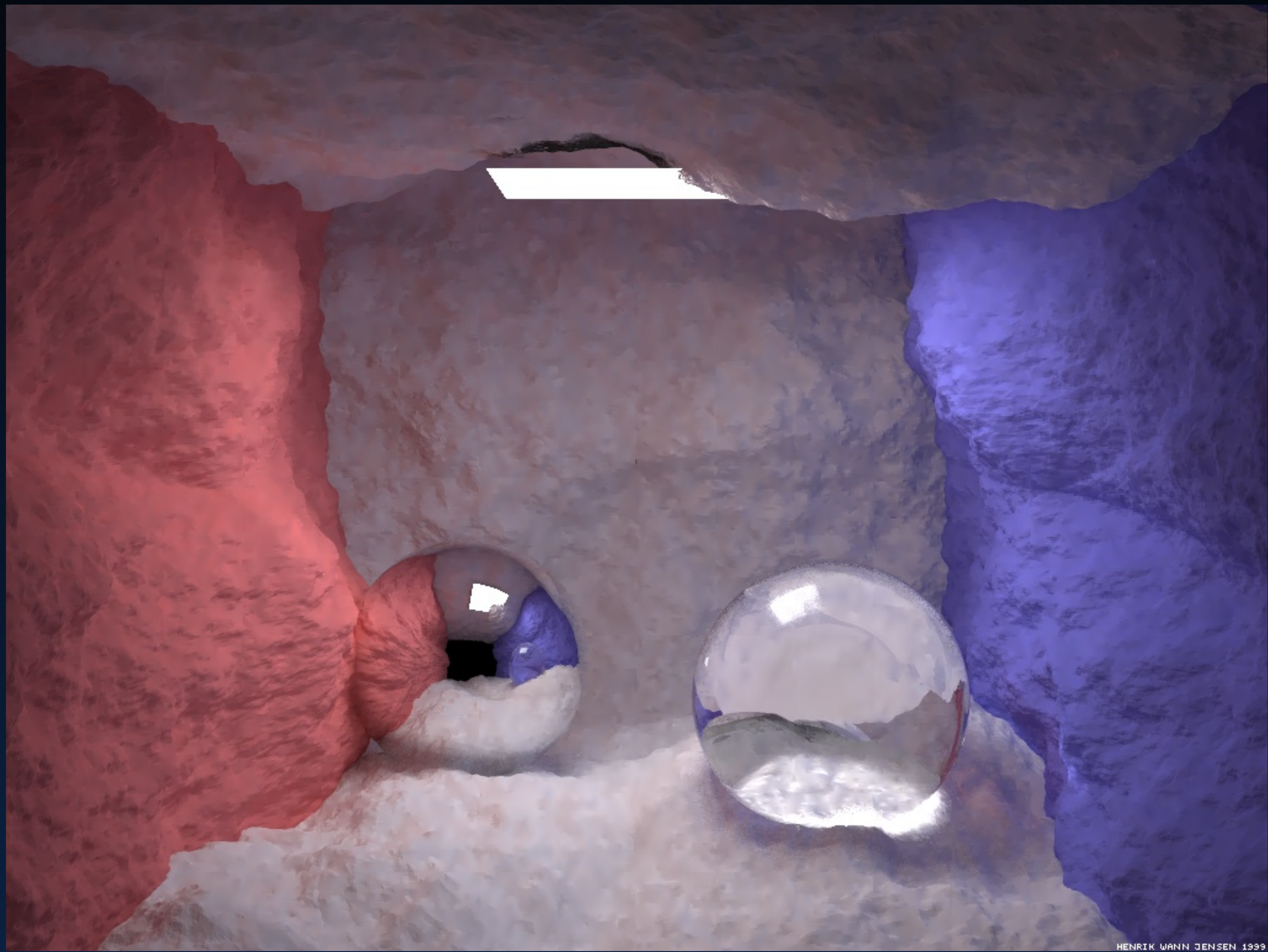
200000 global photons, 50000 caustic photons

Box: Global Photons



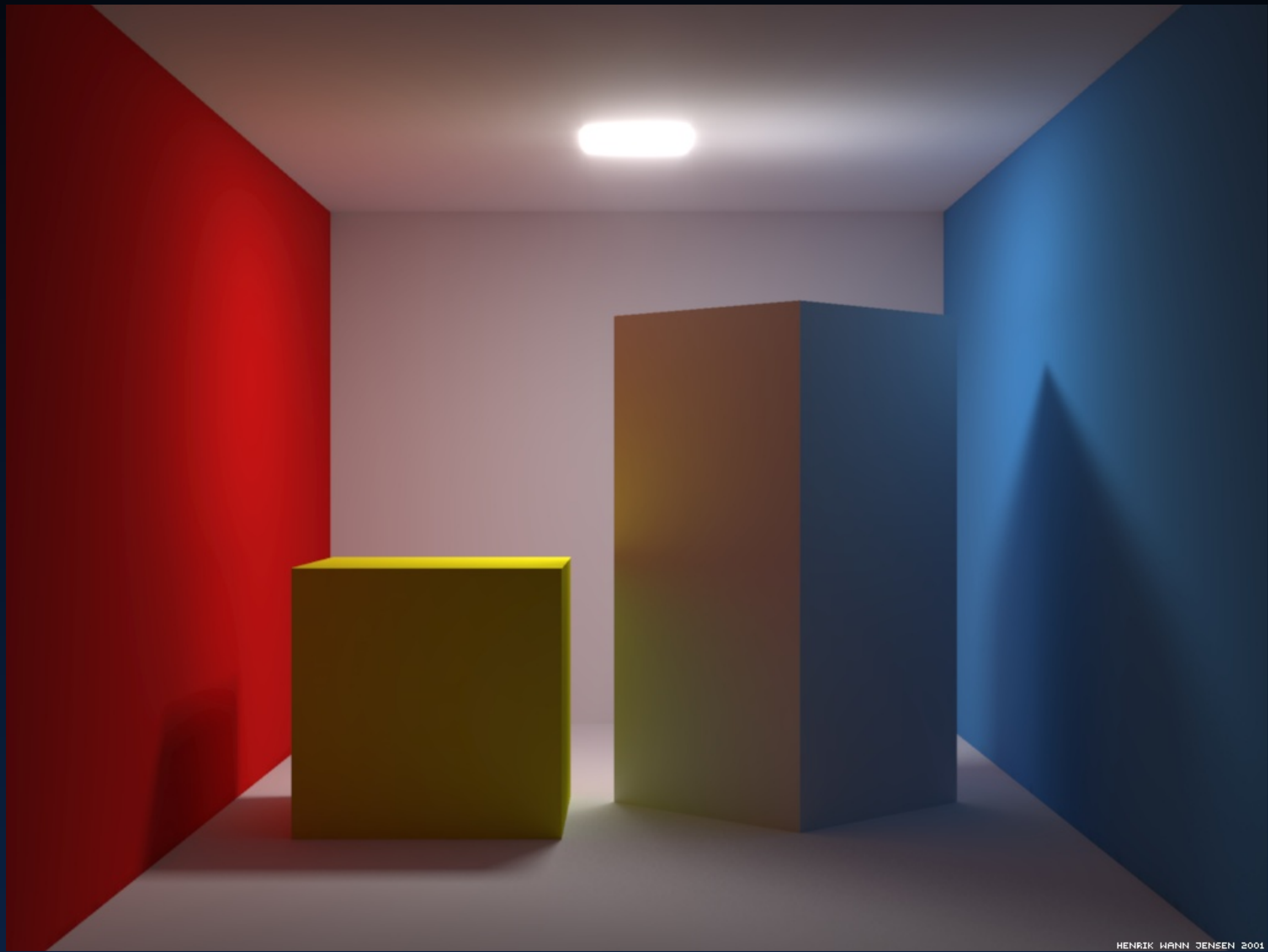
200000 global photons

Fractal Box



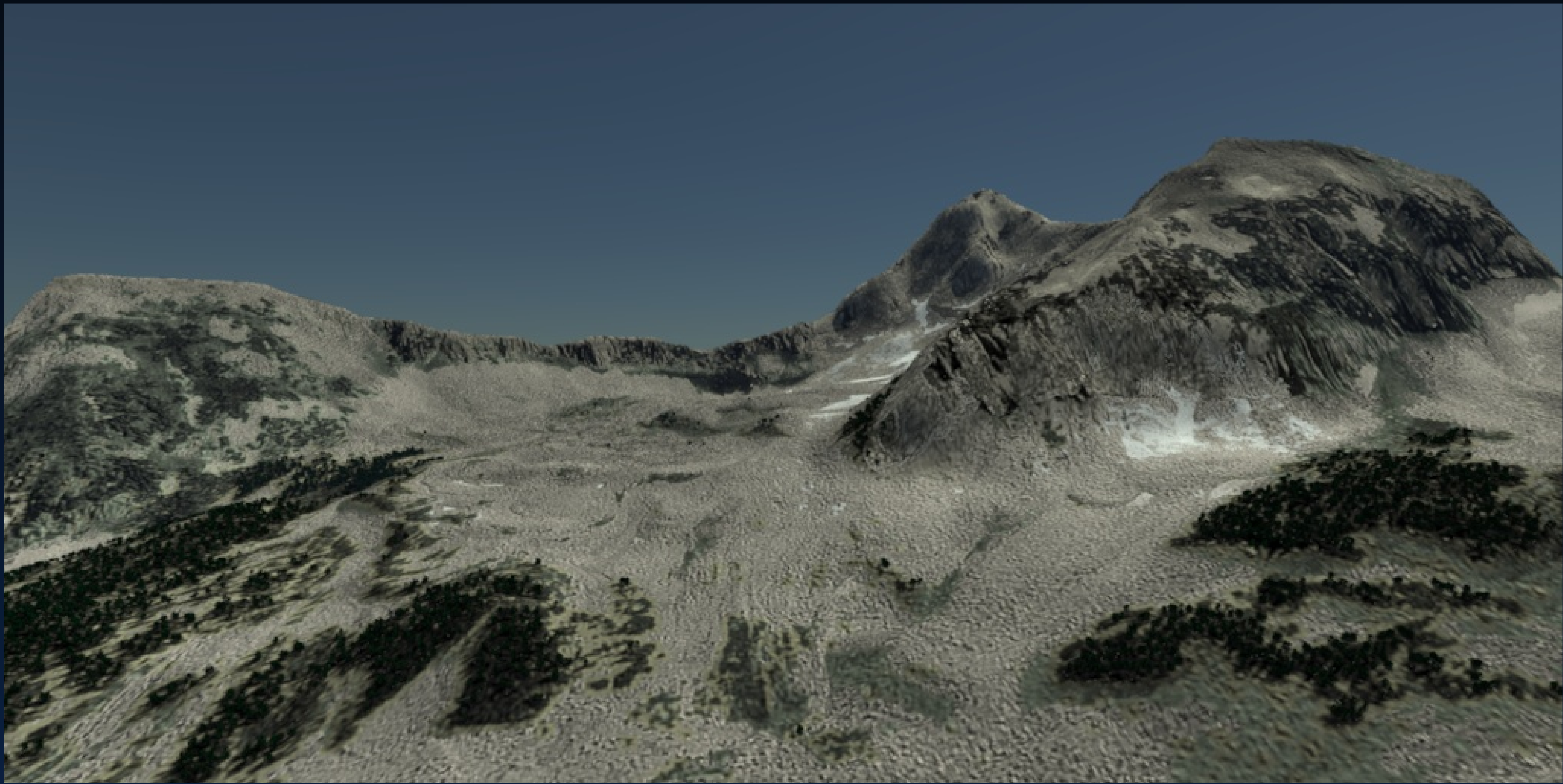
200000 global photons, 50000 caustic photons

Cornell Box



Indirect Illumination

Little Matterhorn

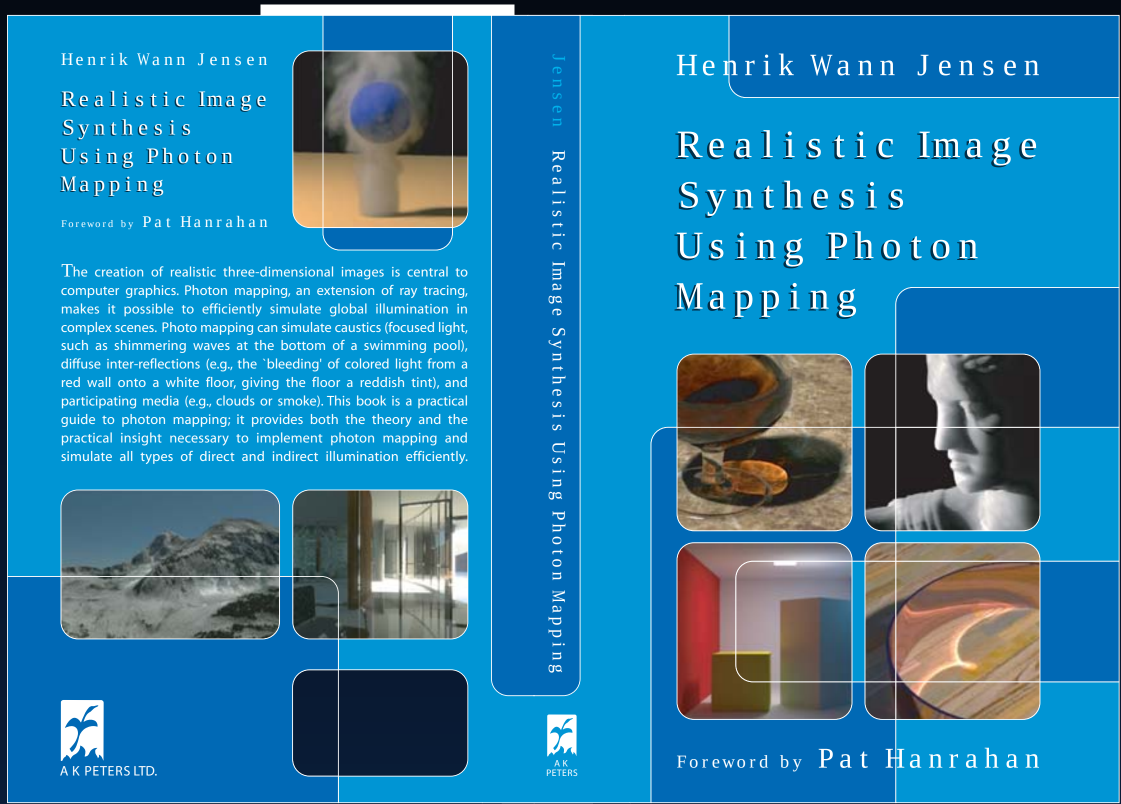


Mies house (swimmingpool)

Mies house (3pm)

Mies house (6pm)

More Information



http://graphics.stanford.edu/~henrik
henrik@graphics.stanford.edu