Introduction to Radiometry and Photometry

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Radiometry

- The goal of a global illumination algorithm is to compute a steady-state distribution of light in a scene
- To compute this distribution, we need an understanding of the physical quantities that represent light energy
- *Radiometry* is the basic terminology used to describe light

Photons

- The basic quantity in lighting is the photon
- The energy (in Joule) of a photon with wavelength λ is: $q_\lambda = hc/\lambda$
  - c is the speed of light
    - In vacuum, c = 299.792.458m/s
  - h ≈ 6.63×10^{-34}Js is Planck's constant

(Spectral) Radiant Energy

- The *spectral radiant energy,* Q_λ, in n_λ photons with wavelength λ is

$$Q_\lambda = n_\lambda q_\lambda$$

- The *radiant energy,* Q, is the energy of a collection of photons, and is given as the integral of Q_λ over all possible wavelengths:

$$Q = \int_0^\infty Q_\lambda d\lambda$$

Radiant Power or Radiant Flux

- *Radiant flux*, also called *radiant power*, is the time rate flow of radiant energy

$$\Phi = \frac{dQ}{dt}$$

- Flux expresses how much energy (Watts = Joule/s) flows to/through/from an (imaginary) surface per unit time
- For wavelength dependence, *spectral radiant flux* is defined as $\Phi_\lambda = \frac{dQ_\lambda}{dt}$

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Radiant Flux Area Density

- The *radiant flux area density* is defined as the differential flux per differential area dΦ/dA
  - In English: The energy arriving at or leaving a surface over a short interval of time
- Traditionally, radiant flux area density is separated into *irradiance*, E, which is flux arriving at a surface and *radiant exitance*, M, which is flux leaving a surface
  - Radiant exitance is also known as *radiosity*, denoted B

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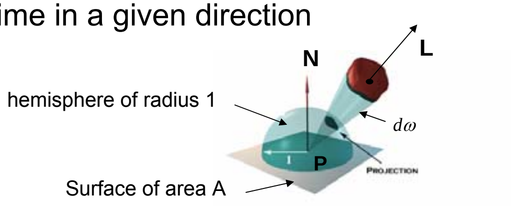
Radiance

- Probably, the most important quantity in global illumination is *radiance*
- Radiance is defined as emitted flux per unit projected area per unit solid angle (W/(steradian*m^2))
- Intuitively, radiance tells us how much energy leaves a small area per unit time in a given direction

$$L = \frac{d^2\Phi}{d\omega dA \cos\theta}$$



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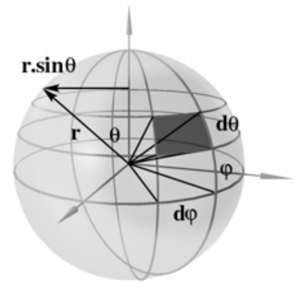
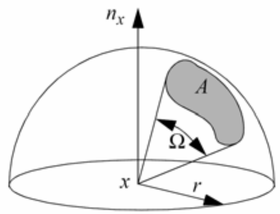
Solid Angle

- *Solid angle* is the measure for ‘angles’ in 3D
  - The unit for solid angle is steradians, ω є [0, 4π]
- The solid angle subtended by an object is defined as the area of the object projected onto a sphere of radius 1 centered at the viewpoint
- The ’size’ of a differential solid angle in spherical coordinates is dω = sinθdθdφ





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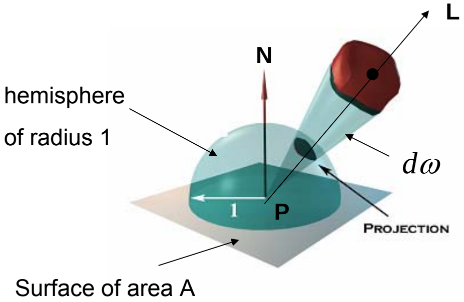
Solid Angle



Back To Radiance

- Radiance is defined as flux per unit projected area per unit solid angle (W/(steradian*m^2))

$$L = \frac{d^2\Phi}{d\omega dA\cos\theta}$$

- An important property of radiance is that, in vacuum, it is constant along a line of sight

Scattering of Light

- When light reaches a surface, it is either scattered or absorbed
  - We assume that the light is scattered immediately after reaching the surface
    - Thus, we ignore fluorescence effects
  - We also assume that light incident at some point also exits at that same point
    - This effectively means no subsurface scattering

BRDF

- A ray of light hits a surface:
  - arriving from a direction $\mathbf{k}_i$,
  - and reflected in the direction $\mathbf{k}_o$
- How much of this light is reflected in the direction $\mathbf{k}_o$?
- This question is answered by the *bidirectional reflectance distribution function*, BRDF

BRDF

- The brdf is a 4 dimensional function defined as

$$f_r(x,k_i,k_o) = \frac{dL_s(x,k_o)}{dE(x,k_i)} = \frac{dL_s(x,k_o)}{L_i(x,\mathbf{k}_i)\cos\theta_i d\omega_i}$$

- BRDF could change over a surface (texture)
- L_s is the outgoing radiance
- L_i is the incoming radiance
- $d\omega_i$ is the differential solid angle associated with the incident direction

BRDF Properties

- A brdf can take on any positive value
  - $f_r(x,\mathbf{k}_i,\mathbf{k}_o) \in [0;\infty[$
- The value of a brdf remains unchanged if the incident exitant directions are interchanged
  - $f_r(x,\mathbf{k}_i,\mathbf{k}_o) = f_r(x,\mathbf{k}_o,\mathbf{k}_i)$
- A physically plausible brdf conserves energy, that is: $\forall \mathbf{k}_i : \int_{\text{all } \mathbf{k}_o} f_r(x,\mathbf{k}_i,\mathbf{k}_o)\cos\theta_o d\omega \le 1$

Directional Hemispherical Reflectance

- Related to the BRDF, we may wish to know exactly how much light is reflected due to light coming from a fixed direction $\mathbf{k}_i$
- This is answered by the *directional hemispherical reflectance*, $R(\mathbf{k}_i)$, given as:

$$R(x,\mathbf{k}_i) = \int_{\text{all } \mathbf{k}_o} f_r(x,\mathbf{k}_i,\mathbf{k}_o)\cos\theta_o d\omega$$

Example

- A *Lambertian* surface is an idealized diffuse surface with a constant brdf, $f_r = c$

$$R(x,\mathbf{k}_i) = \int_{\text{all } \mathbf{k}_o} c\cos\theta_o d\omega$$

$$= \int_0^{2\pi}\int_0^{\pi/2} c\cos\theta\sin\theta d\theta d\phi$$

$$= \pi c$$

- So, for a perfectly reflecting lambertian surface, we have $f_r = 1/\pi$, and if $R(x,\mathbf{k}_i)=r$, $f_r = r/\pi$

The Rendering Equation

- Consider again the brdf: $f_r(x,\mathbf{k}_i,\mathbf{k}_o) = \dfrac{dL_s(x,\mathbf{k}_o)}{L_i(x,\mathbf{k}_i)\cos\theta_i d\omega_i}$
- Rearranging the terms, we get

$$dL_s(x,\mathbf{k}_o) = f_r(x,\mathbf{k}_i,\mathbf{k}_o)L_i(x,\mathbf{k}_i)\cos\theta_i d\omega_i$$

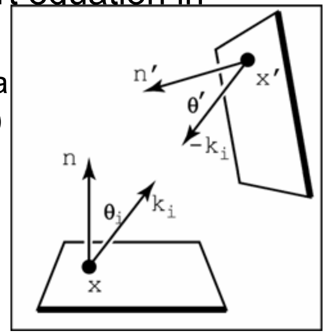
- Integrating over the entire hemisphere, we get the reflected radiance

$$L_s(x,\mathbf{k}_o) = \int_\Omega f_r(x,\mathbf{k}_i,\mathbf{k}_o)L_i(x,\mathbf{k}_i)\cos\theta_i d\omega_i$$

  - This is known as *the rendering equation*
  - For translucent objects, we need the lower hemisphere as well

Alternate Transport Equation

- The rendering equation describes the reflected radiance due to incident radiance on the entire hemisphere
- Sometimes we'll need the transport equation in terms of surface radiance only
  - Because radiance is constant along a straight line, the field radiance $L_i(x,\mathbf{k}_i)$ is equal to the surface radiance from some surface: $L_i(\mathbf{x}, \mathbf{k}_i) = L_i(\mathbf{x'}, -\mathbf{k}_i)$
  - The solid angle subtended by a
  - Surface is $d\omega = \dfrac{dA\cos\theta'}{|\mathbf{x}-\mathbf{x}'|^2}$



Alternate Transport Equation

- Putting this together, we get

$$L_s(\mathbf{x},\mathbf{k}_o) = \int_{\text{all }\mathbf{x}'} \frac{f_r(x,\mathbf{k}_i,\mathbf{k}_o)L_s(\mathbf{x}',\mathbf{x}-\mathbf{x}')v(\mathbf{x},\mathbf{x}')\cos\theta_i\cos\theta' dA}{|\mathbf{x}-\mathbf{x}'|^2}$$

  - Where v(**x**, **x**') is a visibility term, equal to 1 if **x** and **x**' are mutually visible and 0 otherwise
  - $K_i = \overline{x'x}$

- Integral equation: to be solved