A Model of How Interreflections Can Affect Color Appearance

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Received 3 August 1999; accepted 21 October 1999

Abstract: Most studies of surface color appearance have ignored 3-D illumination phenomena such as shadows and interreflections. These phenomena must affect surface color appearance to some extent, but without a model we cannot make claims about how great these effects really are. In this article, we introduce a simple model of how interreflections can affect color appearance. We consider a family of ideal surfaces, namely spherical concavities dug out of a ground plane, and we assume that these surfaces are illuminated by a uniform, diffuse light source. Under such assumptions, the color signal reflected from a surface is described by a simple mathematical model. We use this model and CIELAB coordinates to perform a parametric study of how the lightness, hue, and chroma of the reflected color signal vary with the concavity aperture. We find that interreflections can significantly affect color appearance of spherical concavities, but only if the surface has high lightness. © 2000 John Wiley & Sons, Inc. Col Res Appl, 26, S218–S221, 2001

Key words: color appearance; interreflections; mutual illumination; radiosity equation; diffuse lighting; spherical concavity

INTRODUCTION

The color appearance of a surface depends on both the spectrum of the illuminant and the spectral reflectance of the surface. Most studies of color appearance of surfaces address a scenario in which the surface is planar. In this case, if the surface has spectral reflectance S(λ) and the ambient source has a spectral radiance E(λ), then the color signal $c(\lambda)$ reflected from the surface is

$$c(\lambda) = E(\lambda)S(\lambda). \tag{1}$$

When the surface is nonplanar, Eq. (1) no longer applies. For example, consider a folded surface such as a drapery. Although the reflectance may be uniform over the surface, the color signal reflected from the surface is not uniform. There are two main factors here. First, because of shading and shadowing, the light reaching a point on the surface depends on the surface geometry, for example, whether the point lies on a hill or a valley.[1] Second, points on the surface may illuminate each other via interreflections.[2] Our goal in this article is to investigate the significance of these factors in determining color appearance.

MODEL

Interreflections are described mathematically by the radiosity equation.[3] For a complex scene, the radiosity equation must be solved using numerical methods. In this article, we consider a special case in which the radiosity equation has a closed form solution. This is the case of a spherical concavity of Lambertian reflectance. A spherical concavity is defined by taking the intersection of a hollow sphere with a plane and removing the cap of the sphere above the plane (see Fig. 1).

Spherical concavities are convenient to consider, because they have a closed form solution to the radiosity equation. Moreover, it has been shown that this closed form solution is an excellent approximation for more general surfaces

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Contract grant sponsor: Alexander von Humboldt Research Fellowship

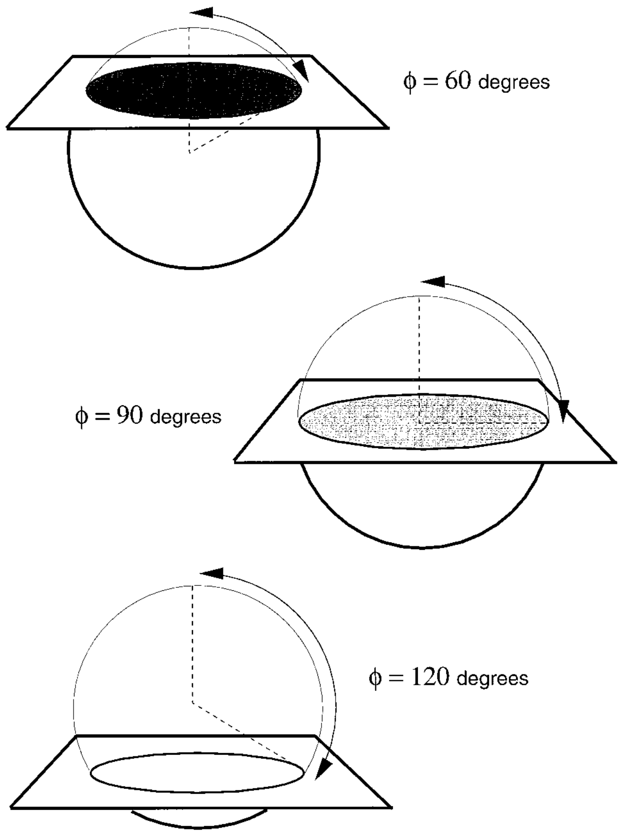

FIG. 1. Three spherical concavities defined by angles ϕ = 60°, 90°, and 120°, where ϕ is the angle between the vertical direction and the direction of the boundary of the concavity, measured from the center of the sphere. Under diffuse lighting, a smaller angle ϕ produces a darker concavity because of shadowing.

such as terrains,[4] and so conclusions about color appearance that are based on spherical concavities can be generalized to these surfaces as well. The closed form solution is as follows. If a spherical concavity of spectral reflectance S(λ) is illuminated by a uniform hemispheric diffuse light source of spectral radiance E(λ), then the color signal reflected from the spherical concavity has the following form[5,6]:

$$c(\lambda,\ \phi) = E(\lambda)S(\lambda)\left(\frac{\dfrac{1-\cos\phi}{2}}{1-\left(\dfrac{1+\cos\phi}{2}\right)S(\lambda)}\right). \qquad (2)$$

The numerator of the fraction on the right represents shadowing effects. It takes values from 0–1 as ϕ goes from 0–180°. The denominator represents a nonlinear gain, which is due to interreflections. This term takes values from 1–S(λ) to 1 as ϕ goes from 0–180°. We see that, for a given ϕ, the color signal depends linearly on the illuminant spectrum E(λ), but nonlinearly on the reflectance spectrum S(λ).

To explore how color appearance depends on interreflections, we consider a family of spherical concavities of reflectance S(λ), excavated from a white ground plane having uniform spectral reflectance of 90%. We fix the illuminant spectrum to the standard CIE D65, and assume that the visual system is adapted to the color signal reflected from the white ground plane. Given this adapted state, we compute the CIELAB coordinates[7] of the family of colored concavities.

Recall that CIELAB is a three-dimensional, percepually uniform color space with coordinates L^*, a^*, b^*. The coordinate L^* represents lightness. The coordinate a^* represents red vs. green. The coordinate b^* represents blue vs. yellow. CIELAB is usually conceived in cylindrical coordinates where lightness L^* is the axis of the cylinder, chroma C^* is the perpendicular distance from the axis of the cylinder, and hue h is an angle between 0–360°.

RESULTS

Figure 2, upper row, shows three different sets of four spectral reflectance functions S(λ). The left and middle columns represent surfaces of high lightness ($L^* > 70$), and the right column represents a surface of medium lightness ($L^* \approx 50$). The left column corresponds to pink surfaces, the middle column to light blue surfaces, and the right column to a middle blue surface.

Figure 2, middle row, shows plots of hue h vs. lightness L^* for angles ϕ ranging from 30–180°. Each plot has four pairs of curves, where each pair consists of a solid curve and a dashed curve. There is one pair of curves for each of the reflectance functions shown in the corresponding column of the upper row. For each pair, the solid curve represents the locus of h and L^* values calculated using Eq. (2), that is, considering shadows and interreflections. The dashed curve represents the locus that is calculated using shadows alone, that is, ignoring the denominator in Eq. (2). The distance Δ between corresponding points on the solid and dashed curves is, thus, the effect of interreflections alone.

Note that for the pink and light blue surfaces, the hue angle is not constant, but rather varies with the shape angle ϕ. The reason is that, as ϕ decreases and the concavity becomes deeper, the average number of reflections within the concavity increases. The hue varies with ϕ, because each reflection modifies the color signal.

Figure 2, bottom row, shows plots of chroma C^* vs. lightness L^*. For each column, the chroma C^* is approximately zero for curve pairs numbered 1 out of 4, because the reflectance spectrum (top row in Fig. 2) is nearly flat, that is, the surface material is near achromatic. For pairs numbered 2, 3, and 4, the surface is chromatic ($C^* > 0$) and the C^* vs. L^* curves show an enormous difference between the solid and dashed cases. This difference is precisely the effect of interreflections. For example, for a pink hemispheric concavity ($\phi = 90°$), interreflections change the color signal by nearly 20 CIELAB units. A sampling of the interreflection effects in shown in Table I for the pink surfaces.

The nonlinearity of the dashed curve is due both to the nonlinear mapping from *XYZ* values to LAB, and to the nonlinearity in the denominator of the right-hand side of Eq. (2). When one replots the curves of Fig. 2, bottom row, in *XYZ* space, one obtains similar behavior qualitatively (not shown).

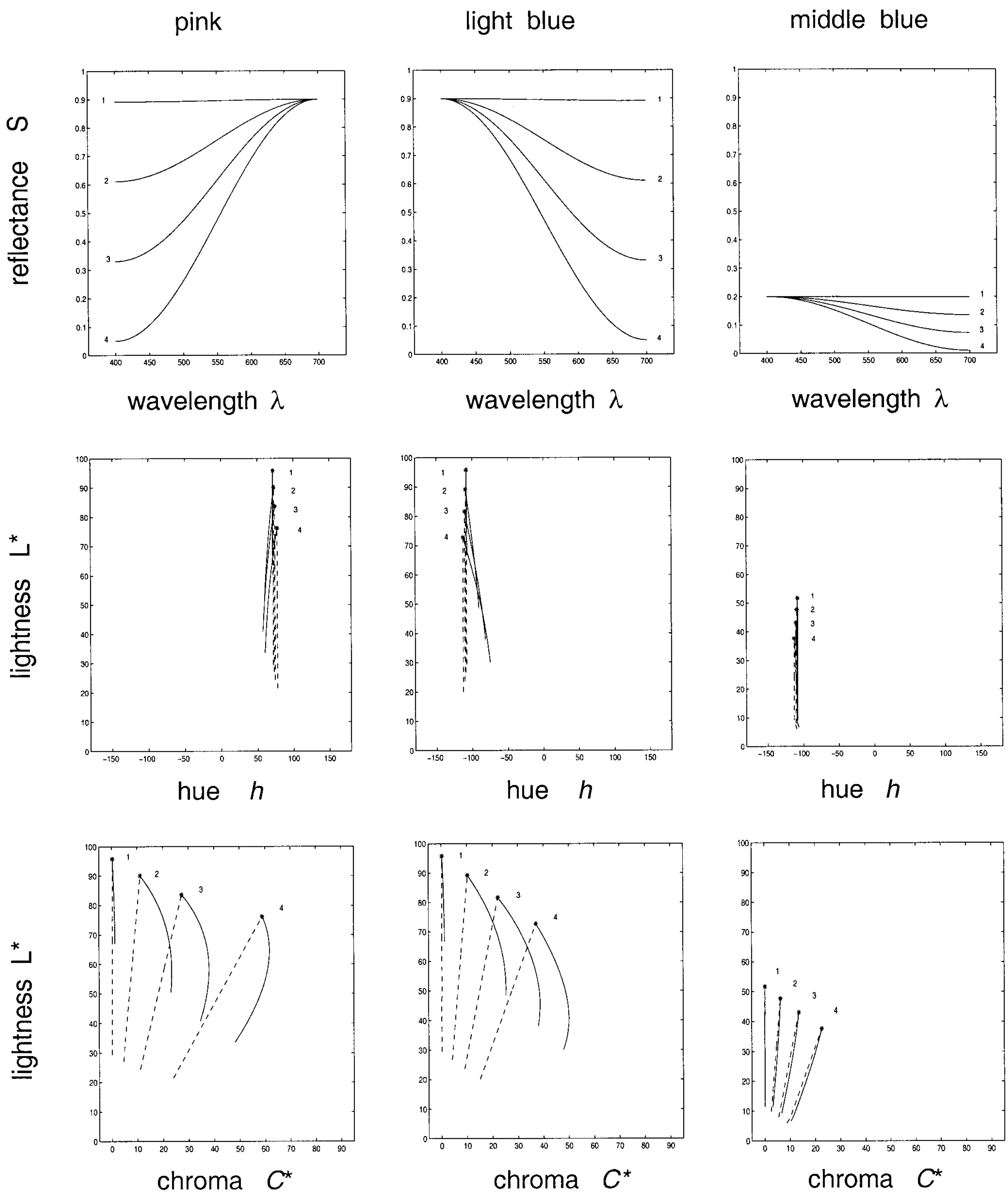

FIG. 2. (Upper row) Spectral reflectance functions S(λ). (Middle and bottom rows) Solid curves show locus of values calculated using both shadows and interreflections. Dashed curves show locus of values calculated using shadows only.

Each dashed and solid curve pair meet at $\phi = 180°$ and diverge for lower values of ϕ, with the dashed curve being linear. The curves meet again (at the origin) at $\phi = 0°$, where the concavity becomes a hollow (dark) sphere. Note that, in Fig. 2 bottom, the curves do not meet at $\phi = 0°$, but this is only because the smallest value of ϕ shown is 30°.

Finally, consider surfaces of middle lightness (Fig. 2, right column). Here interreflections have almost no effect, because

TABLE I. CIELAB distances ϕ between corresponding points of the dashed and solid curves of Fig. 2, bottom left.

ϕ	Pink 1	Pink 2	Pink 3	Pink 4
150	2	2	2	2
120	9	8	8	9
90	18	17	17	19
60	28	27	26	32
30	29	29	30	38

the solid and dashed curves are nearly identical. The reason for the lack of effect is that a surface of middle lightness has a physical reflectance of only 20% (according to CIELAB, for example), and so most of the light reflected from a given concavity is from the first bounce. Interreflections, by definition, concern only the second bounce and beyond.

CONCLUSION

For light colored surfaces ($L^* > 70$) under diffuse illumination, interreflections can have a significant effect on color appearance. For example, in a deep concavity, the effect can be over 30 CIELAB units. For surfaces of low or medium lightness ($L^* < 50$), interreflections have almost no effect on color appearance.

Further studies are needed to examine how interreflection effects are manifest in perception, in particular, how changes in color appearance that are due to interreflections are interpreted by the visual system. Several studies have begun to address this question,[8-10] but more work is needed.

ACKNOWLEDGMENTS

This research was supported by an Alexander von Humboldt Research Fellowship to M. Langer.

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