Modeling Relative Positioning of Handwritten Patterns

Adrien DELAYE, Sébastien MACE and Eric ANQUETIL
INSA de Rennes, Avenue des Buttes de Coësmes, F-35043 Rennes
UMR IRISA, Campus de Beaulieu, F-35042 Rennes
Université Européenne de Bretagne, France
{adrien.delaye, sebastien.mace, eric.anquetil}@irisa.fr

Abstract. In this paper we present a new generic method for modeling and evaluating the relative positioning of handwritten patterns. Thanks to a morphological approach it handles the variability of handwritten shapes and providing a flexible fuzzy description of positioning relations. We propose a way to automatically learn such spatial relation models from samples, and prove that resulting models can be fruitfully used for pattern recognition purpose.

1. Introduction

For recognition and analysis of complex handwritten patterns such as mathematical expressions, diagrams, Chinese characters or even complete structured documents, it is of major interest to model and evaluate the positioning of sub-patterns with respect to each other. Since handwritten patterns are subject to variability and imprecision that affects jointly the shapes and positioning of their sub-components, the actual handwritten shapes should be considered for modeling relative positioning. As demonstrated in (Bloch & Ralescu, 2003), morphological approaches (i.e. that consider the objects shapes) are suited to describe ambiguous positioning relations and to handle correctly objects with concavities, contrarily to methods where the objects are reduced to representative points: centroid methods, bounding box based methods, etc. As an illustration, figure 1(a) presents an ideal positioning template for two sub-components of a composed Chinese character (Wang & Fan, 2001), where the area with a grey background depicts the position of the reference component, and the white area depicts the position of the second one. Figure 1(c) and 1(d) show the large distortions in shapes and relative positioning that can appear for different handwritten samples of the same character, in comparison with theoretical well-formed printed sample 1(b), and thus the need for consideration of the handwritten shapes.

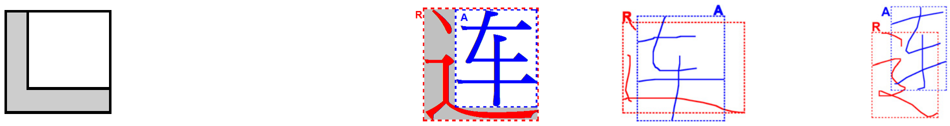


Figure 1. **(a)** template Character "连": **(b)** printed **(c)** handwritten **(d)** handwritten

Interestingly, this problem has not been fully addressed and to our knowledge no generic method was proposed for modeling relative positioning of handwritten patterns. The work presented in (Zhang & al., 2005) is typical of existing methods: authors define a fuzzy logic partition of the plane for handling ambiguities, so as to help recognition of handwritten equations. They define a priori domain-dependant areas: *subscript*, *inline*, etc., and have to manually set associated thresholds, hence a lack of genericity and no easy extension to other domain.

In this paper, we introduce a new generic method for modeling and evaluation of relative positioning of handwritten patterns that handles efficiently the variability and the imprecision of handwriting. We rely on the fuzzy relative positioning approach based on mathematical morphology as introduced by Bloch for directional relative positioning (Bloch 1999).

Our contribution is to exploit Bloch's morphological description for providing an intuitive modeling of positioning relations and to offer an associated automatic learning scheme. We show the resulting positioning models to be satisfactory in the sense that they visually fit the intuition very well, and further prove that they can be used for improving recognition of composed handwritten patterns. Qualitative evaluation is presented on several samples of Chinese patterns, while quantitative experiments are lead on editing gestures, accentuation and punctuation symbols of Latin characters, thus demonstrating the genericity of the method.

The paper is organized as follows. In the next section, we describe the fuzzy relative positioning of graphical objects, as introduced in (Bloch 1999). Then we present in details the extension we propose for description and automatic learning of relative positioning models for handwritten characters. The final section gives experimental results that validate the approach and demonstrate its usefulness for classification.

2. Directional Fuzzy Relative Positioning

The work of Bloch (Bloch 1999) provides a way to describe *directional* fuzzy relative positioning of graphical objects, in the image domain. It was later adapted to the context of online handwritten strokes (Bouteruche & al., 2006), which is also the applicative context we focus on in our experiments. The principle is to describe how an object stands in a given direction with respect to a reference object. A *fuzzy landscape* is first defined around the reference object: it is a fuzzy set that assigns to any point of the plane a membership degree describing how well the point satisfies the considered spatial relation. Then, the object to be positioned relatively to the reference is embedded in the fuzzy landscape in order to evaluate how well it globally matches with the spatial relation.

2.1 Directional Fuzzy Landscape Definition

Defining a directional fuzzy landscape consists in designing a fuzzy set that represents the directional relation characterized by direction α relatively to the reference object R, for any point of the plane. The design of associated fuzzy membership function is based on the concept of smallest deviation angle.

For a point P of the plane, we consider the point Q in R so that the direction QP is as close to α as possible. This leads to the smallest deviation angle for P as expressed by equation [1] and illustrated in figure 2.

$$\beta_{\min}(P)=\min_{Q\in R}\left(\beta(P,Q)\right) \qquad [1]$$

$$\text{with} \quad \begin{cases}\beta(P,Q)=\arccos\left(\vec{u}_{QP}\,.\,\vec{u}_{\alpha}\right)\\ \beta(P,P)=0\end{cases}$$

$$\text{where } \vec{u} \text{ denote unit vectors}$$

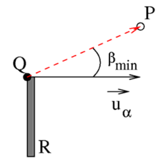
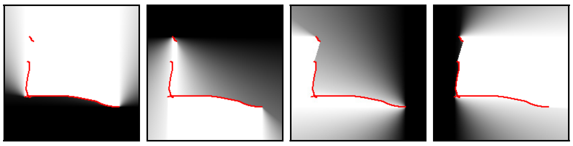


Figure 2. Smallest deviation angle

The actual fuzzy membership function is then simply defined as a linear decreasing function of the smallest deviation angle at the point P, resulting in a membership degree between 0 and 1 as given by equation [2].

$$\mu_{\alpha}(R)(P)=\max\left(0,1-\frac{2\beta_{\min}(P)}{\pi}\right) \qquad [2]$$



(a) up **(b)** down **(c)** left **(d)** right

Figure 3. Directional fuzzy landscapes

Figure 3 shows four fuzzy landscapes defined around the same reference pattern with respect to the four main directions: *up*, *down*, *left*, and *right*. The pictures were simply obtained by assigning to each pixel a grey level proportional to the membership degree of the point to the corresponding landscape, from 0 to 1.

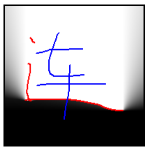
2.2 Comparing Objects with the Fuzzy Landscape

Once the fuzzy landscape is defined, there are several ways to determine the degree to which a given pattern A matches the corresponding positioning with respect to the reference object. Bloch proposes three measures of *mean degree*, *necessity degree*, and *possibility degree*, defined by three following equations. Figure 4 shows the degrees obtained by applying these measures on a given pattern and fuzzy landscape relative to "*up*" direction.

$$M_{\alpha}(R)(A)=\frac{1}{|A|}\sum_{P\in A}\mu_{\alpha}(R)(P) \qquad [3]$$

$$\mathrm{N}_{\alpha}(R)(A)=\inf_{P\in A}\mu_{\alpha}(R)(P) \qquad [4]$$

$$\Pi_{\alpha}(R)(A)=\sup_{P\in A}\mu_{\alpha}(R)(P) \qquad [5]$$



$M_{up}(R)(A)$	0.95
$N_{up}(R)(A)$	0
$\Pi_{up}(R)(A)$	1

Figure 4. Directional position evaluation

Although the *mean* measure better describes the global adequacy of the pattern A with respect to the landscape, *necessity* and *possibility* also carry useful information about the "worst" and "best" points of A.

Since its definition adapts to the shape of reference object, the fuzzy description landscape is likely to handle the variability, and its fuzziness can stress the imprecision of handwriting. The description finally fits the intuition and offers an easy way to evaluate the positioning of patterns relatively to a given direction.

3. Extension to Generic Fuzzy Positioning Models

From this description of directional positioning, we propose to build a relative positioning model as a fuzzy landscape built by intersecting several directional fuzzy landscapes (intersection processed with a fuzzy t-norm operator, for example *product*). Considering the positioning template of figure 1(a), describing a model where a pattern should be written "*on the right of*" and "*above*" the reference, we can model it as $\mu_{ru}=\mu_{right}\cap\mu_{up}$. Figure 5 shows resulting fuzzy landscapes and corresponding mean adequacy scores for patterns of figures 1(c) and 1(d). The obtained model fits the intuition very well and handles correctly the uncertainty of positioning.

Figure 5. **(a)** $M_{ru}(R)(A)=0.82$ **(b)** $M_{ru}(R')(A')=0.89$

However, description of directional relations is not always easy to formulate when the relation is more subtle than the one above. As an example, relative positioning of basic strokes in Chinese characters can have some intricate positioning relations. If we want to model the position of the stroke *A* relatively to reference *R* in the Chinese sub-component “刂”, by looking at handwritten samples from figure 6, a human expert may say that “*A is on the* left of R”, but this would lead to a very rough description where any point of the half-plane on the left of R has a high membership degree.

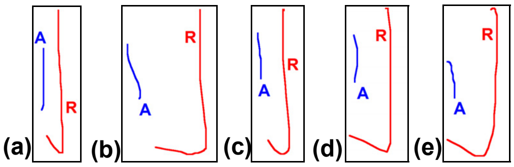


Figure 6. Five handwritten samples of character “刂”

Table 1. Necessity and possibility degrees for **6.(a)**

α	up	down	left	right
$N_\alpha(R)(A)$	0.88	0.77	1.00	0.00
$\Pi_\alpha(R)(A)$	0.98	0.90	1.00	0.00

In this case, it would be beneficial if the model could also capture how the positioning of *A* is with respect to *R* in the *up* direction as well, but this is not something that can be described in an easy way.

We introduce here a new general definition of positioning model that takes into account “*to what extent” A* can be in each of the four main directions with respect to *R*. Specifying *“to what extent”* consists in defining some variability in the membership degrees admitted in each directional relation. Since this is not possible to linguistically describe this variability, we also provide a simple way to learn it from training data.

3.1 Describing Fuzzy Directional Variability

Adequacy degree for a directional positioning usually varies for different points of a stroke *A* as well as for different instances of *A*, and we want to determine this variability. We propose to define fuzzy sets representing the admitted values of membership degrees for each of the four directional positions.

A simple way to estimate the bounds of variability is to consider the *necessity* and *possibility* measures (see equations [4] and [5]), since they describe the lowest and highest membership degrees for points of *A*. They can also be interpreted as boundaries of minimal deviation angle with respect to the considered direction. Table 1 presents degrees computed from the sample of figure 6(a), considering the four directions. According to horizontal directions, *A* is obviously on the left of *R*, and not on its right at all. More interestingly, measures obtained for *up* and *down* provide precise information about positioning: membership takes values in a rather narrow range of degrees in both the *up* and *down* fuzzy directional landscapes.

Given a set of samples labelled as *R* and *A* that follow a positioning model *M* (e.g. 5 couples of fig. 6), it is quite easy to learn directional variability by computing mean necessity and possibility values, and associated variance. We can then represent *M* by 4 fuzzy sets such as the one defined by equation [6] and depicted in fig. 7.

For a directional degree $x = \mu_\alpha(R)(P), x \in [0,1]$

$$\phi_\alpha^M(x) = \begin{cases} 1/(1+d_Q(x,\overline{N}_\alpha)) & if \quad x < \overline{N}_\alpha \\ 1 \quad if \quad \overline{N}_\alpha < x < \overline{\Pi}_\alpha \\ 1/(1+d_Q(x,\overline{\Pi}_\alpha)) \; if & x > \overline{\Pi}_\alpha \end{cases} \qquad [6]$$

where $\overline{N}_\alpha, \overline{\Pi}_\alpha$ are the mean necessity and possibility values from training data, and $d_Q(x,Z)$ is the Mahalanobis distance from x to Z, taking into account the estimated variance around the Z mean value.

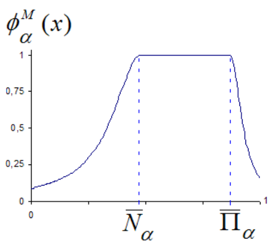


Figure 7. variability in α direction

3.2 Filtered Fuzzy Landscape

Once we modeled the admitted variability of directional membership degrees for a given model, we are able to define directional fuzzy landscapes that precisely reflect this model. This is done by applying the directional variability fuzzy sets as filters on the simple directional fuzzy landscapes, according to equation [7].

$$\mu_\alpha^M(R)(P) = \phi_\alpha^M(\mu_\alpha(R)(P)) \qquad [7]$$

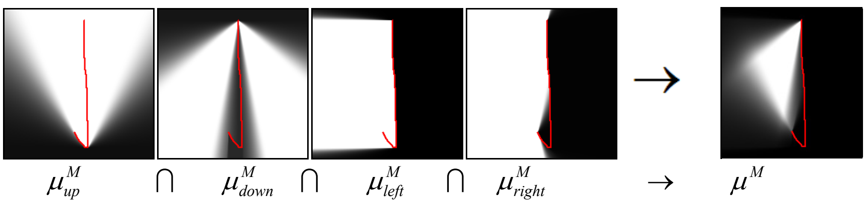


Figure 8. Filtered directional landscapes and global landscape

The 4 directional landscapes can be intersected, resulting in a global fuzzy landscape that represents the complete accurate positioning model (see figure 8). The fuzzy landscape finally defines a limited area where the stroke A is expected to be written given the shape and position of reference R. This area is both small and accurate, showing the relevance of the proposed method. Likewise, evaluation of adequacy to a modeled positioning relation can then be computed with the mean membership value of the points of A to this global fuzzy landscape. Figure 9 presents 5 fuzzy landscapes obtained by applying the same learnt model on the 5 reference strokes of samples from figure 6, and associated mean adequacy degree.

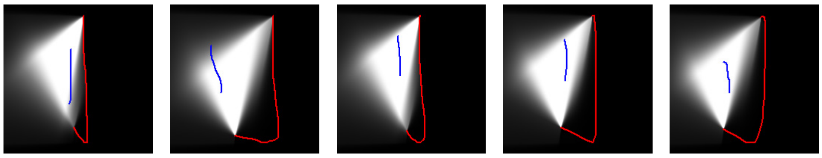


Figure 9. **(a)** 0.90 **(b)** 0.83 **(c)** 1.00 **(d)** 1.00 **(e)** 0.99

The images confirm that the positioning model is able to adapt to the shape of the reference stroke, handling properly the variability of handwriting, and the results of positioning scores show that the associated strokes are largely well located in the expected areas according to the model (white area of the fuzzy landscapes).

4. Experiments

In order to validate the method, we conducted experiments on a set of online handwritten graphic gestures. The data set consists of 18 classes of gestures drawn relatively to a handwritten Latin letter: it includes punctuation symbols, accentuation, and editing gestures (space, caret return, etc.), written on a PDA by 15 different writers. Because some classes have very similar shapes (e.g. acute, apostrophe and coma), this dataset is relevant for evaluating effectiveness of positioning models (Bouteruche & al., 2006). We used SVM classifiers to compare efficiency of different spatial relation features, combined with 9 simple shape features. Classifiers training implied 3,704 samples from 10 writers and test was processed on 1,821 samples from 5 other writers.

In *test 0*, we obtain a baseline rate by trying to recognize gestures without any spatial positioning description. For *test 1* we used 4 positioning features as the mean adequacy degrees (see eq. [3]) according to the 4 directions under the Bloch's definition. Finally, in *test 2* we learnt one positioning model for each class following the method exposed in section 3, and used mean adequacy degrees to the 18 models as positioning features for SVM input. Figure 10 shows samples from the "apostrophe" gesture class and associated fuzzy landscapes obtained by applying the same positioning model on 5 different reference letters. Table 2 presents the classification rates on test dataset for the three sets of features. It appears that our modeling of relative positioning permits to enhance the classification performance in comparison with *test 1*, with error reduction of about 16,45%.

Table 2. Comparison of positioning features

test	recognition rate
test 0	53,52 %
test 1	95,82 %
test 2	96,40 %

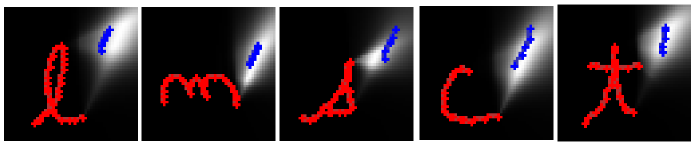


Figure 10. Positioning model for "apostrophe"

5. Conclusion

In this paper, we propose a new method for modeling and evaluating relative positioning of handwritten patterns that can adapt to the actual shapes of patterns for handling variability of handwriting, by combining morphological approach and fuzzy representation. We generalize Bloch's directional positioning description for modeling more precise positioning relations and offer a way for automatically learn the models. Results show that models finally fit the intuition very well on different types of handwritten patterns, and numerical experiments proved that the models are beneficial to statistical recognition of composed patterns. In our future work, we aim at using this new positioning modeling method for structural recognition of online handwritten Chinese characters, for example by integrating them in a fuzzy inference system such as in (Delaye & al., 2008).

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