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Nonlinear system fault detection and isolation based on bootstrap particle filters

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***Abstract*— A particle filter based method for nonlinear system fault detection and isolation is proposed in this paper. It is applicable to quite general stochastic nonlinear dynamic systems in discrete time. The main result consists of a new particle filter algorithm, derived from the basic bootstrap particle filter, and capable of rejecting a subset of the faults possibly affecting the considered system. Fault isolation is then achieved by the evaluation of the estimated likelihoods related to the designed filters.**

***Index Terms*— fault detection and isolation, stochastic nonlinear system, particle filter.**

I. INTRODUCTION

Due to the increasing complexity of engineering systems, fault detection and isolation (FDI) are becoming an important issue for the design of fault tolerant systems and for maintenance optimization. Early studies on FDI have been mainly focused on linear systems [1], [2], [3], [4], [5]. Recently, nonlinear system FDI have become an active research topic, mainly concerning deterministic approaches based on the techniques of nonlinear observers and differential algebra. See, e.g., [6], [7], [8], [9], [10].

In this paper, stochastic nonlinear dynamic systems are considered. The proposed FDI method is based on a new *particle filter* algorithm (see [11] for a general introduction to particle filters), applicable to quite general nonlinear dynamic systems in a stochastic framework. Stochastic uncertainties are naturally addressed by particle filters. Moreover, unknown disturbances can also be rejected with the proposed new particle filter algorithm. More formally, let us consider stochastic nonlinear dynamic systems in discrete time subject to faults, in the form of

$$x_{k+1} = f(x_k, u_k) + v_k^x + \sum_{i=1}^{n_f} \psi^i(x_k, u_k) q_k^i \quad \text{(1a)}$$

$$\eta_k = g(x_k, u_k) + v_k^\eta \quad \text{(1b)}$$

where $x_k \in \mathbb{X} \subset \mathbb{R}^{n_x}$ is the state vector, $u_k \in \mathbb{R}^{n_u}$ the input vector, $\eta_k \in \mathbb{R}^{n_\eta}$ the output vector, $v_k^x \in \mathbb{R}^{n_x}$ and $v_k^\eta \in \mathbb{R}^{n_\eta}$ are state and output noises, and the terms $\psi^i(x_k, u_k)q_k^i$ represent faults affecting the system. The noises v_k^x and v_k^η are assumed to be white with known probability density functions (PDF) $p_x(v_k^x)$ and $p_\eta(v_k^\eta)$. The PDF of the initial state x_0 is assumed to be $p_0(x)$. The assumptions about the nonlinear functions f and g will be stated in Section III-B. The sequence $q_k^i \in \mathbb{R}$ is zero when the i-th fault is absent, and it is an arbitrary unknown non zero sequence when the fault has occurred. The nonlinear functions $\psi^i(x_k, u_k) \in \mathbb{R}^{n_x}$ structurally specify the assumed faults.

The problem of fault detection is to detect the presence of any faults $\psi^i(x_k, u_k)q_k^i$ from the available input-output signals u_k and η_k. This is a relatively easy task, since for a filter designed by ignoring all the faults $\psi^i(x_k, u_k)q_k^i$, any anomaly in its functioning reveals the violation of the fault-free hypothesis. The main difficulty of FDI is in the step of fault *isolation*: decide which subset of the possible faults is likely to be present. This problem is known as the "fundamental problem of residual generation" for residual based FDI approaches, originally defined for linear systems [12].

The problem formulated above is similar to the one considered in [8], apart from the differences between continuous time and discrete time and also between deterministic and stochastic systems. A solution based on the deterministic approach has been presented in [8], whereas the particle filter based method proposed in the present paper is proposed in a stochastic framework with more general structural assumptions.

Remark that, in this paper, each fault is assumed to be an *arbitrary* unknown sequence q_k^i. This assumption has the advantage to be general, embracing different practical situations. However, this property is at the price of an important sensor requirement: in order to completely isolate all the considered faults, the number of output sensors n_η must be greater than or equal to the number of faults n_f. This sensor requirement can be weakened if some specific assumption is made about the fault profiles. For instance, when faults are modeled as parametric changes (rare jumps of constant parameter values), a particle filter based approach with weaker sensor requirement has been developed in [13].

The application of particle filters to change detection problems has been studied by some authors. The computation of generalized likelihood ratio (GLR) tests with the aid of particle filters has been reported in [14] which is also reviewed in [15]. A statistic for slow change detection has been proposed in [16]. These works do not consider fault *isolation*, which is the main topic of the present paper.

The paper is organized as follows. A short introduction to the basic particle filter is given in Section 2. The new particle filter algorithm for fault isolation is presented in Section 3. A numerical example is given in Section 4. Some conclusions are drawn in Section 5.

II. Short introduction to the basic bootstrap particle filter algorithm

This section gives a brief informal introduction to the basic particle filter algorithm, also known as *bootstrap filter* [17], applicable to the *fault-free* system considered in this paper. It is intended for readers not familiar with particle filters. For more complete and general presentations, the readers are referred to [11], [18].

Let us consider system (1) in the fault-free case which is rewritten as

$$x_{k+1} = f(x_k, u_k) + v_k^x \tag{2a}$$
$$\eta_k = g(x_k, u_k) + v_k^\eta \tag{2b}$$

Denote by D_k the input-output data observed up to the time instant k:

$$D_k = \{(u_i, \eta_i) : \ i = 1, \ldots, k\}$$

The filtering problem is to estimate the distribution of the state vector at each instant k, based on the data observed up to instant k, or more precisely, to estimate the conditional PDF $p(x_k|D_k)$. In general, no accurate finite dimensional filter exists for nonlinear systems, even if the noises are assumed to be Gaussian. The basic idea of particle filters is to approximate the PDF of x_k at each instant k with the sum of (a large number of) Dirac functions, and to make them evolve at each time instant based on the latest observed data. Each Dirac function used in the PDF approximation is called a *particle*.

To start the particle filter at the initial instant $k = 0$, randomly draw M points in $\mathbb{R}^{n_x}$ following the assumed PDF $p_0(\cdot)$ of the initial state vector. Let us denote these M points with the vectors $\xi_0^j \in \mathbb{R}^{n_x}$, $j = 1, \ldots, M$, then $p_0(\cdot)$ is approximated by

$$p(x_0|D_0) \approx \frac{1}{M}\sum_{j=1}^{M}\delta(x_0 - \xi_0^j)$$

Recursively, at each instant $k \geq 0$, with

$$p(x_k|D_k) \approx \frac{1}{M}\sum_{j=1}^{M}\delta(x_k - \xi_k^j)$$

already estimated, the distribution of x_{k+1} is first predicted with the state equation (2a), leading to an approximation of the PDF $p(x_{k+1}|D_k)$. For this purpose, each particle ξ_k^j, for $j = 1, \ldots, M$, is propagated following the state equation (2a) to the position $f(\xi_k^j, u_k)$ and perturbed by a random vector γ_k^j drawn following the state noise PDF $p_x(\cdot)$, yielding

$$\xi_{k+1|k}^j = f(\xi_k^j, u_k) + \gamma_k^j$$

Then

$$p(x_{k+1}|D_k) \approx \frac{1}{M}\sum_{j=1}^{M}\delta(x_{k+1} - \xi_{k+1|k}^j)$$

Now the data observed at instant $k+1$ is used to estimate $p(x_{k+1}|D_{k+1})$. According to the Bayes rule, each particle $\xi_{k+1|k}^j$ is weighted by its likelihood w_{k+1}^j based on the output equation (2b):

$$w_{k+1}^j = p_\eta\Big(\eta_{k+1} - g(\xi_{k+1|k}^j, u_k)\Big)$$
$$S_{k+1} = \sum_{j=1}^{M} w_{k+1}^j$$
$$p(x_{k+1}|D_{k+1}) \approx \frac{1}{S_{k+1}}\sum_{j=1}^{M} w_{k+1}^j\delta(x_{k+1} - \xi_{k+1|k}^j)$$

In order to approximate $p(x_{k+1}|D_{k+1})$ with M equally weighted particles, randomly draw M points following the *discrete* probability distribution

$$P(x = \xi_{k+1|k}^j) = \frac{w_{k+1}^j}{S_{k+1}}, \quad j = 1, \ldots, M$$

The resulting points, noted as $\xi_{k+1}^j \in \mathbb{R}^{n_x}$ for $j = 1, \ldots, M$, is then used to make the approximation

$$p(x_{k+1}|D_{k+1}) \approx \frac{1}{M}\sum_{j=1}^{M}\delta(x_{k+1} - \xi_{k+1}^j)$$

The algorithm then goes to the next iteration with k increased by 1.

Summary of the algorithm:

Particle initialization. Draw M points $\xi_0^j \in \mathbb{R}^{n_x}$ for $j = 1, \ldots, M$ following the initial state PDF $p_0(\cdot)$.

Particle propagation. At each instant $k \geq 0$, draw M points $\gamma_k^j \in \mathbb{R}^{n_x}$ following the state noise PDF $p_x(\cdot)$ and compute

$$\xi_{k+1|k}^j = f(\xi_k^j, u_k) + \gamma_k^j$$

Particle weighting. Compute the likelihood of each particle $\xi_{k+1|k}^j$ and their sum:

$$w_{k+1}^j = p_\eta\Big(\eta_{k+1} - g(\xi_{k+1|k}^j, u_k)\Big)$$
$$S_{k+1} = \sum_{j=1}^{M} w_{k+1}^j$$

Particle Resampling. Draw M points $\xi_{k+1}^j \in \mathbb{R}^{n_x}$ following the discrete probability distribution

$$P(x = \xi_{k+1|k}^j) = \frac{w_{k+1}^j}{S_{k+1}}, \quad j = 1, \ldots, M$$

and make the approximation

$$p(x_{k+1}|D_{k+1}) \approx \frac{1}{M}\sum_{j=1}^{M}\delta(x_{k+1} - \xi^j_{k+1})$$

III. Particle filter for fault isolation

For the purpose of fault isolation, the method proposed in this paper is to design several particle filters, each assuming a different subset of the possible faults formulated in system (1), while ignoring the complementary subset of faults. These particle filters then work in parallel. By evaluating the estimated likelihood associated to each filter, the subset of faults most likely present in the system is decided, accomplishing the fault isolation task.

A. Basic idea

The key issue is thus how to design a particle filter with an assumed subset of faults present in the system. For the sake of simplicity in our presentation, let us assume a single fault $\psi^i(x_k, u_k)q^i_k$. The designed filter then should detect all the possible faults, except the i-th one. Such a filter is said to be *rejecting the i-th fault*. The generalization to filters rejecting more faults is straightforward.

Remark that each term $\psi^i(x_k, u_k)q^i_k$ in system (1) can either represent a fault or an unknown disturbance. In the latter case, the designed filter rejecting q^i_k allows to make fault detection *robust* to the unknown disturbance. The numerical example given in Section IV is such a robust fault detection problem.

For fault isolation to make sense, there must be at least two possible faults. The number of output sensors must be greater than or equal to the number of faults for complete fault isolation, therefore $n_\eta \geq 2$.

Note on notations. Let $y_k \in \mathbb{R}$ denote a chosen component of η_k, and z_k its complementary part. In other words, when the components of η_k are appropriately arranged,

$$\eta_k = \begin{bmatrix} y_k \\ z_k \end{bmatrix}$$

The index i in the term $\psi^i(x_k, u_k)q^i_k$ will be dropped for lighter notation.

With these new notations, the problem of designing a particle filter rejecting the fault $\psi(x_k, u_k)q_k$ is considered for the system

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) + v^x_k + \psi(x_k, u_k)q_k && \text{(3a)} \\ y_k &= g(x_k, u_k) + v^y_k && \text{(3b)} \\ z_k &= h(x_k, u_k) + v^z_k && \text{(3c)} \end{aligned}$$

The noises v^y_k and v^z_k are assumed independent of each other and have respectively the PDF $p_y(v^y_k)$ and $p_z(v^z_k)$.

Now the problem is to design a particle filter capable of estimating the conditional PDF $p(x_k|D_k)$, despite the presence of the fault $\psi(x_k, u_k)q_k$. It is similar to the unknown-input observer in the deterministic framework. See, e.g., [5].

When trying to apply the basic particle filter algorithm recalled in the previous section to the present case, the tricky point is the Particle propagation step, due to the presence of $\psi(x_k, u_k)q_k$ in the state equation (3a).

If some knowledge about the statistic distribution of the fault q_k is available, the fault can be treated in a way similar to the state noise v^x_k, then the application of the basic particle filter algorithm is relatively trivial. In this paper, no statistic distribution of the fault q_k is assumed. In this case, one can still try to apply the basic particle filter algorithm by assuming uniform distribution of q_k over a *large* range, but it will lead to poor results because of the largely scattered particles generated by the particle propagation step with the large uniform distribution of q_k. In order to avoid this problem, the idea of this paper is to use part of the output equation, say (3b), to compensate the lack of knowledge about the fault q_k in the state equation.

More precisely, at instant k, in order to propagate the particle ξ^j_k to $\xi^j_{k+1|k}$, after having drawn $v^x_k \in \mathbb{R}^{n_x}$ following $p_x(\cdot)$ and $v^y_{k+1} \in \mathbb{R}$ following $p_y(\cdot)$, it is possible to solve the $n_x + 1$ equations

$$\xi^j_{k+1|k+} = f(\xi^j_k, u_k) + v^x_k + \psi(\xi^j_k, u_k)q_k \tag{4}$$

$$y_{k+1} = g(\xi^j_{k+1|k+}, u_{k+1}) + v^y_{k+1} \tag{5}$$

for the n_x+1 unknowns $\xi^j_{k+1|k+} \in \mathbb{R}^{n_x}$ and $q_k \in \mathbb{R}$. Notice that in the above equations the notation of $\xi^j_{k+1|k+}$ is used instead of $\xi^j_{k+1|k}$, because its computation requires u_{k+1} and y_{k+1} (but not z_{k+1}).

If this solution is successful, then the resulting particles $\xi^j_{k+1|k+}$ can be weighted with the remaining output observation z_{k+1}, as if they were the particles $\xi^j_{k+1|k}$ in the basic particle filter algorithm. However, the equations (4) and (5) may not allow to solve for $\xi^j_{k+1|k+}$, because the output equation (5) may not carry the information missing in the state equation (4). Let us illustrate this problem with a simple linear system example.

Consider the linear system

$$\begin{aligned} x_{k+1} &= Ax_k + v^x_k + \psi q_k && \text{(6a)} \\ y_k &= c\,x_k + v^y_k && \text{(6b)} \end{aligned}$$

with $n_x = 2$, $\psi = [0\ 1]^T$ and $c = [1\ 0]$. The fault q_k affects the second state equation only. For this example, the equations (4) and (5) become

$$\xi^j_{k+1|k+} = A\,\xi^j_k + v^x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} q_k \tag{7}$$

$$y_{k+1} = [1\ 0]\ \xi^j_{k+1|k+} + v^y_{k+1} \tag{8}$$

The first component of $\xi^j_{k+1|k+}$ can be easily computed, since q_k does not affect the first state equation. The problem is how to compute its second component. Obviously, equation (8) does not provide any information about the second component of $\xi^j_{k+1|k+}$. It is thus impossible to solve these equations for $\xi^j_{k+1|k+}$.

When such a problem occurs, a solution may exist by making use of observations at instant $k+2$ or even later, if the extra delay of the filter[1] is acceptable in practice. Let us still use the above linear system example to illustrate the idea.

The fact that equation (8) does not provide useful information is due to the equality $c\,\psi = 0$. It follows from the system model (6) that

$$\begin{aligned} y_{k+2} &= c\,x_{k+2} + v^y_{k+2} \\ &= cAx_{k+1} + cv^x_{k+1} + v^y_{k+2} + c\,\psi q_{k+1} \end{aligned}$$

With randomly drawn values of v^x_{k+1} and v^y_{k+2}, equation (8) can be replaced by

$$y_{k+2} = cA\xi^j_{k+1|k+} + c\,v^x_{k+1} + v^y_{k+2} + c\,\psi q_{k+1} \quad (9)$$

Apparently, now there is a new unknown q_{k+1} in equation (9). Remind that it is the equality $c\,\psi = 0$ that made equation (8) useless. Fortunately, if $c\,\psi = 0$, the unknown q_{k+1} is not really involved in equation (9), which can indeed be used with equation (7) to solve for $\xi^j_{k+1|k+}$, if $cA\psi \neq 0$.

If $cA\psi = 0$ and in the case of $n_x > 2$, the same operation can be repeated by making use of y_{k+3}, and so on. If the matrix pair (A, c) is assumed observable, then the observability matrix is of full rank. Then there must exist an integer $i < n_x$ such that $cA^i\psi \neq 0$. Notice that here the observability is a sufficient, but not necessary condition.

B. Assumptions

Now let us more formally state the conditions required by the proposed particle filter algorithm for FDI.

Assumption 1: Consider the system equations (3). At any instant k, there exists an integer $i \geq 0$ such that, for any given value of the state $x_k \in \mathbb{X}$, and for any given values of v^x_j, v^y_j with $j = k, \ldots, k+i+1$, the value of $x_{k+1} \in \mathbb{X}$ can be uniquely determined from the observations u_j, y_j and v^x_j, v^y_j with $j = k, \ldots, k+i+1$. □

The uniqueness of $x_{k+1} \in \mathbb{X}$ is assumed for presentation simplicity. It could be relaxed to a finite number of values.

Assumption 1 ensures the particle propagation from ξ^j_k to $\xi^j_{k+1|k+}$, as intended in (4), and thus the applicability of the proposed FDI algorithm. However, this assumption is not easy to check, because of its generality. The following more restrictive assumption is easier to check and yet covers a large class of nonlinear dynamic systems.

Roughly speaking, if the state noise v^x_k was neglected, the following assumption means that the composite functions g, $g \circ f$, $g \circ f \circ f$, etc., up to $i-1$ levels of composition, are independent of $\psi^T x$, and that the i-th composition depends on $\psi^T x$ in an invertible manner. The formal statement is given in the following.

Assumption 2: Consider the case of constant $\psi(x_k, u_k)$, simply denoted as $\psi \in \mathbb{R}^{n_x}$, then the system model is rewritten as

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) + v^x_k + \psi q_k && (10a) \\ y_k &= g(x_k, u_k) + v^y_k && (10b) \\ z_k &= h(x_k, u_k) + v^z_k && (10c) \end{aligned}$$

The functions $f(x, u)$ and $g(x, u)$ are differentiable in x. Define the notations

$$s_k = \psi^T x_k \quad (11)$$

$$\mathbb{S} = \{s | s = \psi^T x, x \in \mathbb{X}\} \quad (12)$$

where $\mathbb{X}$ is the domain of x_k. For any instant k, assume that there exists an integer $i \geq 0$ such that

$$\frac{\partial y_{k+i+1}}{\partial s_{k+i+1-j}} = 0 \text{ for } j = 0, 1, \ldots, i-1 \quad (13)$$

$$\frac{\partial y_{k+i+1}}{\partial s_{k+1}} \neq 0 \quad (14)$$

hold for all $s_{k+i+1-j} \in \mathbb{S}$, $j = 0, 1, \ldots, i$, with constant sign of $\partial y_{k+i+1}/\partial s_{k+1}$. □

This new assumption can be easily checked in practice with equations (10a) and (10b). Notice that condition (13) means the independence of y_{k+i+1} on $s_{k+i+1-j}$. This assumption ensures the applicability of the proposed FDI algorithm, as well as Assumption 1. This fact is stated in the following proposition.

Proposition 1: Assumption 2 implies Assumption 1. □

Proof of Proposition 1. Some notations need to be introduced first for the proof.

Let μ_l be the collection of the history of u_k, v^x_k, v^x_k *up to* instant l.

Define

$$g_0(x_l, \mu_l) \triangleq g(x_l, u_l) + v^y_l$$

and, recursively, for $j = 1, 2, 3, \ldots$,

$$g_j(x_{l-j}, \mu_l) \triangleq g_{j-1}\left(f(x_{l-j}, u_{l-j}) + v^x_{l-j}, \mu_l\right)$$

Let $\Phi \in \mathbb{R}^{n_x \times (n_x - 1)}$ be a matrix of full column rank such that $\Phi^T\psi = 0$. Remind $s_{l-j} = \psi^T x_{l-j}$ as defined in (11), therefore the information about x_{l-j} can be fully recovered from s_{l-j} and $\Phi^T x_{l-j}$. Since these two "parts" of x_{l-j} need to be distinguished in the following, let us define the new notation $\tilde{g}_j(\cdot, \cdot, \cdot)$ such that

$$g_j(x_{l-j}, \mu_l) = \tilde{g}_j(s_{l-j}, \Phi^T x_{l-j}, \mu_l)$$

Now the preparation of notations is ready.

[1] Strictly speaking, it is no longer a filter, because observations later than $k+1$ are used for the estimation of the state distribution at instant $k+1$.

With the above defined notations, the goal of the proof below is to show

$$y_{k+i+1} = g_i(x_{k+1}, \mu_{k+i+1})$$

which would be combined with (10a) to solve for x_{k+1}. This equation would exactly provide the information missing in (10a), as ensured by condition (14).

By time shifting, equation (10b) is rewritten as

$$y_{k+i+1} = g(x_{k+i+1}, u_{k+i+1}) + v^y_{k+i+1}$$

which, following the definition of g_0, trivially leads to

$$y_{k+i+1} = g_0(x_{k+i+1}, \mu_{k+i+1})$$

Now the question is if $y_{k+i+1} = g_1(x_{k+i}, \mu_{k+i+1})$ holds, and similarly $y_{k+i+1} = g_2(x_{k+i-1}, \mu_{k+i+1})$, and so on.

Recursively, assume that

$$y_{k+i+1} = g_j(x_{k+i+1-j}, \mu_{k+i+1}) \tag{15}$$

holds for some $j < i$, then let us prove the case of $j + 1$.

In (15), when $x_{k+i+1-j}$ is replaced by

$$f(x_{k+i-j}, u_{k+i-j}) + v^x_{k+i-j} + \psi q_{k+i-j}$$

the key issue is to show that the term ψq_{k+i-j} is not really involved. For this purpose, the assumed equation (15) is rewritten as

$$y_{k+i+1} = \tilde{g}_j(s_{k+i+1-j}, \Phi^T x_{k+i+1-j}, \mu_{k+i+1})$$

Remind that $j < i$. Due to condition (13), y_{k+i+1} does not depend on $s_{k+i+1-j}$. Let us mark this fact in the last equation by replacing $s_{k+i+1-j}$ with $\emptyset$. Replace also $\Phi^T x_{k+i+1-j}$ with

$$\begin{aligned}\Phi^T x_{k+i+1-j} &= \Phi^T[f(x_{k+i-j}, u_{k+i-j}) + v^x_{k+i-j} + \psi q_{k+i-j}] \\ &= \Phi^T[f(x_{k+i-j}, u_{k+i-j}) + v^x_{k+i-j}]\end{aligned}$$

where the equality $\Phi^T \psi = 0$ has been used. It then yields

$$y_{k+i+1} = \tilde{g}_j\left(\emptyset, \Phi^T[f(x_{k+i-j}, u_{k+i-j}) + v^x_{k+i-j}], \mu_{k+i+1}\right)$$

which, according to the definition of $\tilde{g}_j$, is rewritten as

$$\begin{aligned}y_{k+i+1} &= g_j(f(x_{k+i-j}, u_{k+i-j}) + v^x_{k+i-j}, \mu_{k+i+1}) \\ &= g_{j+1}(x_{k+i-j}, \mu_{k+i+1})\end{aligned}$$

where the last equality follows the recursive definition of g_j. In the case $j = i - 1$, the equality

$$y_{k+i+1} = g_i(x_{k+1}, \mu_{k+i+1}) \tag{16}$$

is exactly the expected one. It can then be combined with (10a) to solve for x_{k+1}, as ensured by condition (14).

C. Algorithm

The following particle filter algorithm rejecting the fault q_k is stated under Assumption 2 for its easy notations. Its generalization to the case of Assumption 1 is straightforward.

Particle initialization. Draw M points $\xi^j_0 \in \mathbb{R}^{n_x}$ for $j = 1, \dots, M$ following the initial state PDF $p_0(\cdot)$.

Particle propagation. At each instant $k \geq 0$, assume that $g_0, \dots, g_{i-1}$ are independent of $\psi^T x$ and that g_i is invertible with respect to $\psi^T x$. For each particle ξ^j_k, randomly draw $v^x_k, v^x_{k+1}, \dots, v^x_{k+i-1}$ and v^y_{k+i+1}, solve the equations

$$\begin{aligned}\xi^j_{k+1|k+} &= f(\xi^j_k, u_k) + v^x_k + \psi q_k \\ y_{k+i+1} &= g_i(\xi^j_{k+1|k+}, \mu_{k+i+1})\end{aligned}$$

for $\xi^j_{k+1|k+} \in \mathbb{X}$, where g_i is as in (16), μ_{k+i+1} is filled with $v^x_k, v^x_{k+1}, \dots, v^x_{k+i-1}$ and v^y_{k+i+1}. Store the noise realization $v^x_{k+1}, \dots, v^x_{k+i-1}$ for use in the next iteration.

Particle weighting. Compute the likelihood of each particle $\xi^j_{k+1|k+}$ and their sum:

$$\begin{aligned}w^j_{k+1} &= p_z\Big(z_{k+1} - h(\xi^j_{k+1|k+}, u_k)\Big) \\ S_{k+1} &= \sum_{j=1}^{M} w^j_{k+1}\end{aligned}$$

Particle Resampling. Draw M points $\xi^j_{k+1} \in \mathbb{R}^{n_x}$ following the discrete probability distribution

$$P(x = \xi^j_{k+1|k+}) = \frac{w^j_{k+1}}{S_{k+1}}, \quad j = 1, \dots, M$$

and make the approximation

$$p(x_{k+1}|D_{k+1+}) \approx \frac{1}{M}\sum_{j=1}^{M} \delta(x_{k+1} - \xi^j_{k+1})$$

where the notation $p(x_{k+1}|D_{k+1+})$ means the PDF of x_{k+1} conditioned by D_{k+1} and y_{k+1+i}.

D. FDI through estimated likelihood

For the purpose of FDI, several particle filters must run in parallel, each rejecting a different subset of the faults q^i_k assumed in system (1). If one of them works "poorly", then it is decided that the faults *ignored* by this filter has occurred. The "poorness", or rather, "goodness" of each filter is evaluated by the likelihood corresponding to the filter, which is estimated at each time instant k by

$$L_k \approx \frac{1}{M}\sum_{j=1}^{M} w^j_k$$

The estimated likelihood values can be combined inside a sliding time window for more smooth decisions.

IV. NUMERICAL EXAMPLE

Let us borrow the point mass satellite example from [8] with their notations. The point mass system has 4 state variables: the polar coordinates ρ, φ, the radial velocity v and the angular velocity ω, governed by the state equations

$$\begin{aligned}
\dot{\rho} &= v \\
\dot{v} &= \rho\omega^2 - \theta_1 \frac{1}{\rho^2} + \theta_2 u_1 + w \\
\dot{\varphi} &= \omega \\
\dot{\omega} &= -\frac{2v\omega}{\rho} + \theta_2 \frac{u_2}{\rho} + \theta_2 \frac{m}{\rho}
\end{aligned}$$

where θ_1, θ_2 are known constant parameters, u_1, u_2 are radial and tangential thrust controls, m represents the the tangential thrust actuator fault, w is an unknown disturbance (which could also model the radial thrust actuator fault). Let us assume two output sensors measuring ρ and φ with additive Gaussian noises (In [8] one more sensor measuring ω is also assumed). The system is simulated in continuous time in Simulink with $\theta_1 = 0.1$, $\theta_2 = 0.2$, the initial state vector $[2.97, -0.07, 1.81, 0.47]^T$, the constant inputs $u_1 = -3$, $u_2 = -1$, and the disturbance $w(t) = 0.5 \sum_{j=5}^{9} \sin(j0.1t)$. Two scenarios are simulated, the fault-free case ($m(t) \equiv 0$) and the total tangential actuator fault case with $m(t)$ jumps from 0 to 1 at $t = 10$. In each case the input-output signals are sampled with the sampling period $T_s = 0.02$ and with white Gaussian noises of standard deviation 0.001 added to each output.

The differential equations are discretized with the simple Euler method. A particle filter rejecting the disturbance w is then designed, with y corresponding to r and z to φ. For this example, Assumption 2 is satisfied with $i = 1$. The estimated likelihood L_k with $M = 100$ particles is depicted in Figure 1 for each of the two simulated scenarios. In the fault-free case, the likelihood is quite stationary, whereas in the faulty case, after the occurrence of the fault at $t = 10$, the likelihood clearly goes toward zero.

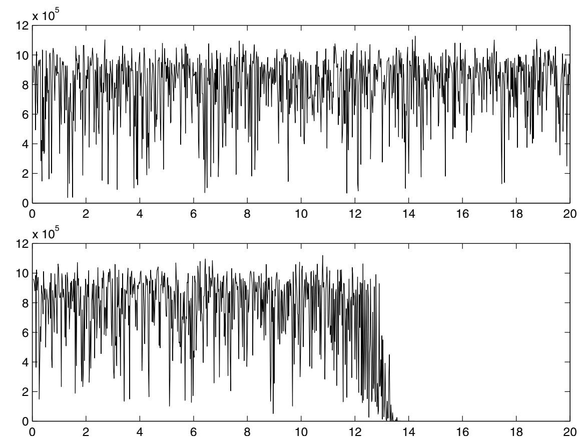

Fig. 1. Estimated likelihood for the fault-free case (top) and for the faulty case (bottom).

V. CONCLUSION

A particle filter based method has been presented in this paper for FDI. Compared to existing deterministic methods, this method naturally deals with stochastic nonlinear dynamic systems, is conceptually simple, and is applicable to quite general nonlinear systems. This generality is at the price of intensive numerical computation, like in most particle filter applications.

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