Particle Filter Track Before Detect Algorithms

Theory and Applications

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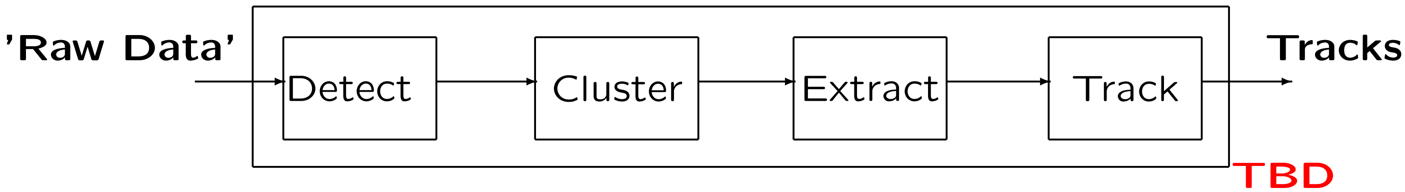
Introduction

Classical vs. TBD



TBD integrates the information over time.

Detection is based on power/energy that has been integrated over time (multiple scans).

Classical tracking : single scan based detection.

* TBD provides higher probability of detection (P_d) at the same level of probability of false alarm (P_{fa})
* TBD circumvents the data association problem.

Introduction

Twofold problem

The TBD problem is twofold:

1. Filtering
2. Detection

Filtering

The System

$$s_{k+1} = f(t_k, s_k, d_k, w_k), \quad k \in \mathbb{N}$$

$$\text{Prob}\{d_{k+1} = j \mid d_k = i\} = [\Pi(t_k)]_{ij}$$

$$z_k = h(t_k, s_k, d_k, v_k), \quad k \in \mathbb{N}$$

Filtering Problem: Determine $p(s_k, d_k | Z_k)$

Basic idea of the particle filter

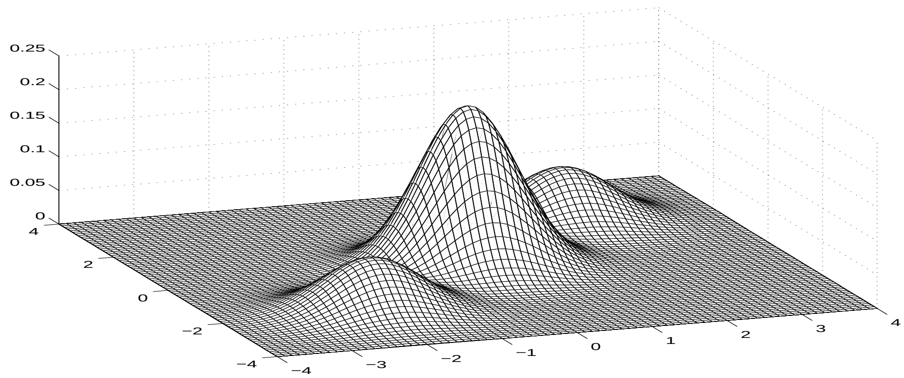
" Describe the a posteriori pdf $p(s_k, d_k|Z_k)$ *by a cloud of* N *particles that propagates in time such that the cloud approximately equals an N-sample drawn from* $p(s_k, d_k|Z_k)$ *"*

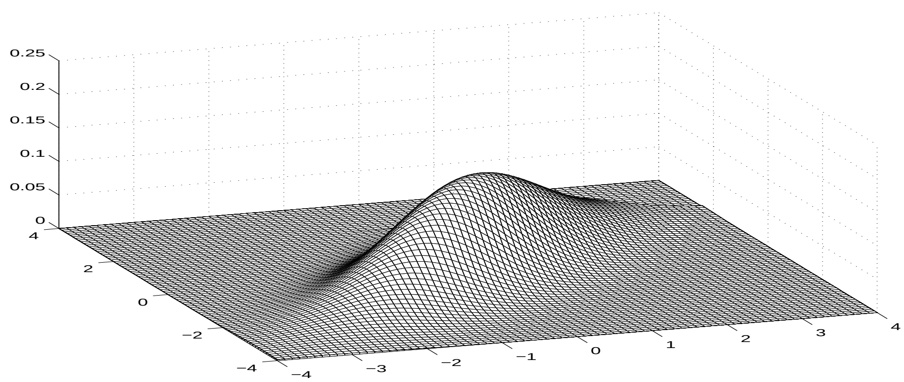
NOTE:

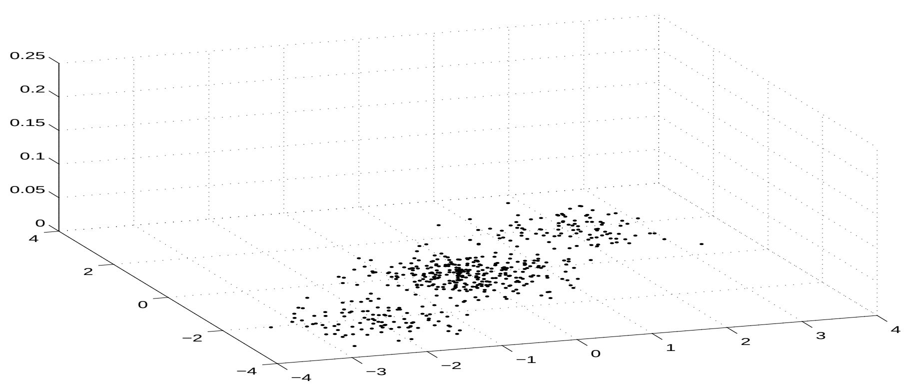
This is more than just a (point) estimate !!!!

Filtering

Kalman vs. PF representation







Filtering

Using a (proper) particle filter on the system:

The following holds

$$\sum_{i=1}^{N} \frac{1}{N}\delta(s - \tilde{s}_k^i) \xrightarrow{a.s.} p(s_k|Z_k)$$

i.e. almost sure convergence...

Popular (point) estimators obtained from particle cloud:

$$\hat{s}_k^{MV} = \int_{\mathbb{R}^n} s_k p(s_k \mid Z_k) ds_k \approx \sum_{i=1}^{N} \frac{1}{N} \tilde{s}_k^i$$

$$\hat{s}_k^{MAP} = \arg \max_{s_k \in \mathbb{R}^n} p(s_k \mid Z_k) \approx s_k^{i^*}$$

where $i^* = \arg\max_i q_k^i$

Detection

Deciding upon presence of target:

Hypothesis testing:

Given two hypotheses

- $\mathcal{H}_0$: no signal present

$$z(j) = v(j), \;\; j = 0, \ldots, k$$

- $\mathcal{H}_1$: signal present

$$z(j) = h(s(j), v(j)), \;\; j = 0, \ldots, k$$

  where $s(k)$ evolves according to dynamical system

Detection

Using particle filter output for detection

Every optimal detector can be expressed in terms of a Likelihood Ratio Test:

$$L(Z(k,l)) \lesseqgtr \tau$$

THEOREM:

$$L(Z(k,l)) = \frac{p(z(k-l+1),\ldots,z(k) \mid \mathcal{H}_1)}{p(z(k-l+1),\ldots,z(k) \mid \mathcal{H}_0)} \approx$$

$$\approx \frac{\prod_{j=k-l+1}^{k}(\sum_{i=1}^{N}\tilde{q}^i(j))}{N^l \prod_{j=k-l+1}^{k} p_v(z(j))}$$

Using particle filter output for detection

Elements of the Proof:

$$p(z(l),\ldots,z(m)\mid\mathcal{H}_0)=\prod_{j=l}^{m}p_v(z_j)$$

$$p(z(l),\ldots,z(m)\mid\mathcal{H}_1)=\prod_{j=l}^{m}p(z(j)\mid Z(j-1))$$

where

$$p(z(j)\mid Z(j-1))=\int_{\mathcal{S}}p(z(j),s\mid Z(j-1))ds$$

$$=\int_{\mathcal{S}}p(z(j)\mid s,Z(j-1))p(s\mid Z(j-1))ds$$

$$=E_{p(s|Z(j-1))}p(z(j)\mid s,Z(j-1))$$

$$\approx\frac{1}{N}\sum_{i=1}^{N}p(z(j)\mid s^i(j))=\frac{1}{N}\sum_{i=1}^{N}\tilde{q}^i(j)$$

Example - Detection

Linear Gaussian scalar system:

$$s(k+1) = s(k) + w(k)$$

$$z(k) = s(k) + v(k)$$

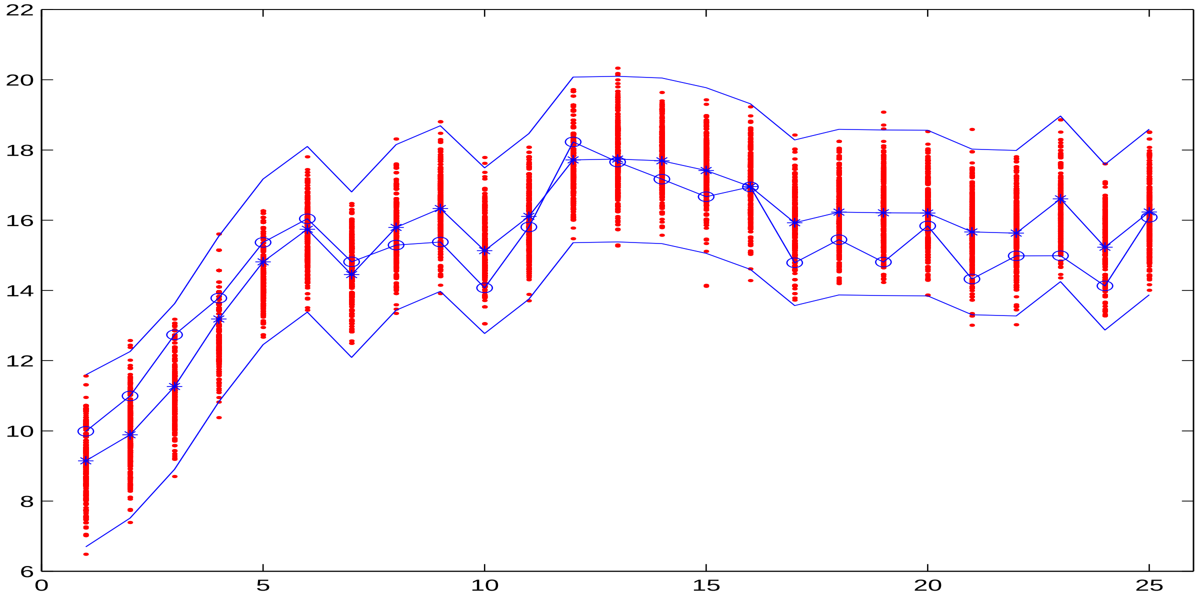
$w(k) \sim N(0,1)$, $v(k) \sim N(0,1)$ and $s(0) \sim N(0,10)$

Data has been generated according to the above model.

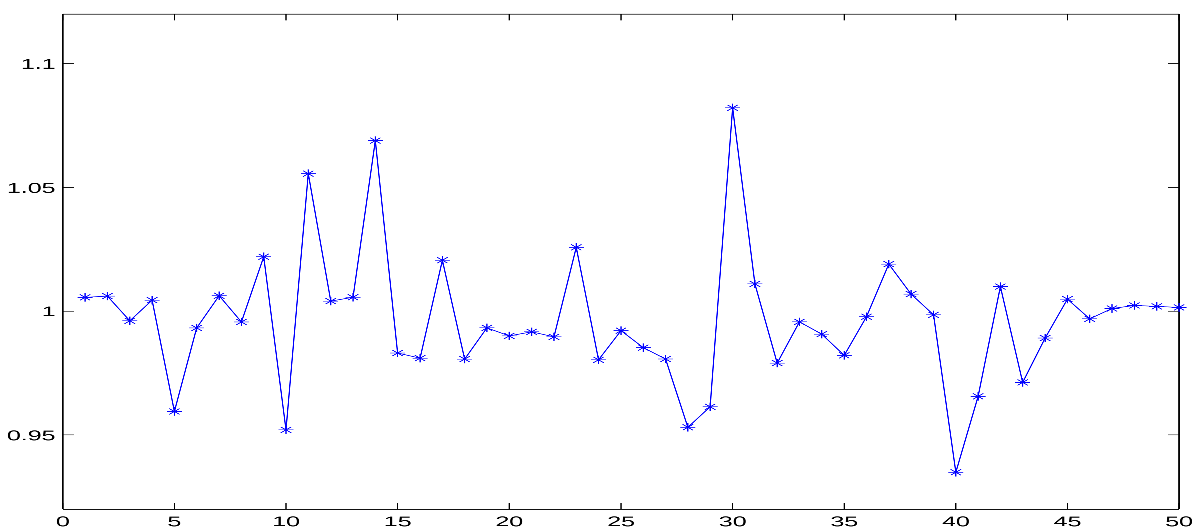
Particle filter solution (200 particles) and the **exact (Kalman) solution** have been calculated.

Example - Detection

True states and estimates



Ratio exact and p.f. likelihood



Example - MTT TBD

The Fighter-Missile Example:

Multi target track before detect application for *small to very small closely spaced targets*. Early detection is crucial.

Modelling details in:

Y. Boers, J.N. Driessen, F. Verschure, W.P.M.H. Heemels and A. Juloski. A Multi Target Track Before Detect Application. *Workshop on Multi Object Tracking*, Madison, WI, June 2003.

Example - MTT TBD

System:

$$s_{k+1} = f(t_k, s_k, d_k) + g(t_k, s_k, d_k) w_k$$

where

$$f(t_k, s_k, d_k) = \begin{pmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} s_k$$

The process noise input model is given by

$$g(t_k, s_k, d_k) = \begin{pmatrix} \frac{1}{2}(\frac{1}{3}a_{x,max})T^2 & 0 \\ 0 & \frac{1}{2}(\frac{1}{3}a_{y,max})T^2 \\ \frac{1}{3}a_{x,max}T & 0 \\ 0 & \frac{1}{3}a_{y,max}T \end{pmatrix}$$

Example- MTT TBD

System:

The discrete mode d_k represents one of three hypotheses (*each have a different measurement equation!!*)

- $d_k = 0$: There is no target present.
- $d_k = 1$: The prime target is present.
- $d_k = 2$: There are two targets present.

Markov process:

$$\Pi(t_k) = \begin{pmatrix} 0.90 & 0.10 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ 0.00 & 0.10 & 0.90 \end{pmatrix}$$

Example - MTT TBD

Simulations

Initially, there is no target present. The first target (fighter: SNR=13dB) appears after 5 seconds ($T = 1$s) and moves at a constant velocity of 200ms^{-1} towards the sensor.

After 20 seconds, a second target (missile: SNR 3dB) spawns from the first and accelerates to a velocity of 300ms^{-1} in 3 scans.

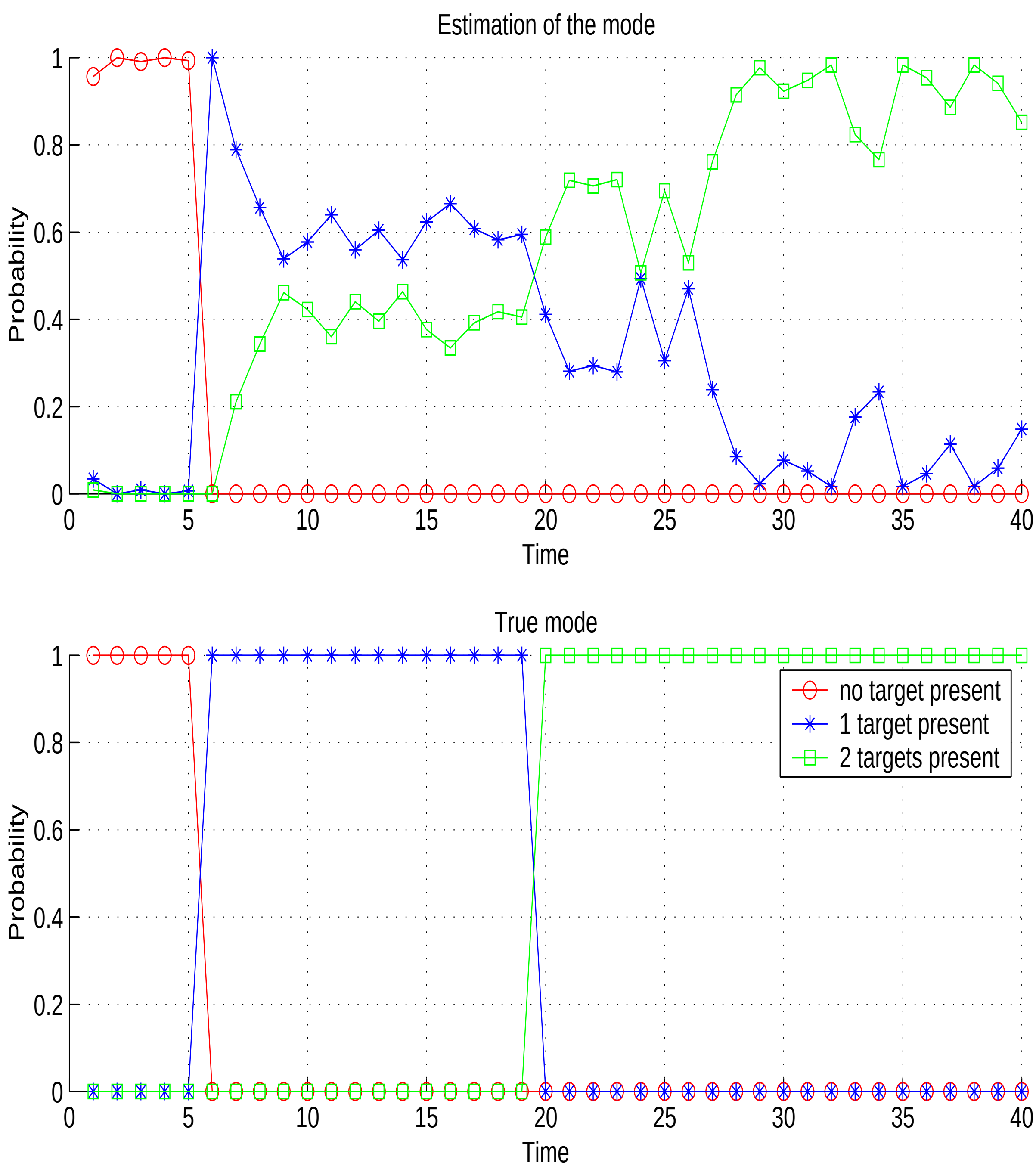
1000 particles have been used in a *'plain vanilla particle filter implementation'*

Example- MTT TBD

Simulations

Matlab movies

Example - MTT TBD



Overview

Related work (co)authored by presenter:

Y. Boers and J.N. Driessen. A Particle Filter Based Detection Scheme. *IEEE Signal Processing Letters*, October, 2003.

Y. Boers, J.N. Driessen, F. Verschure, W.P.M.H. Heemels and A. Juloski. A Multi Target Track Before Detect Application. *Workshop on Multi Object Tracking*, Madison, WI, June 2003.

Y. Boers and J.N. Driessen. Hybrid State Estimation: A Target Tracking Application. *Automatica*, vol. 38, no.12, 2002.

Y. Boers and J.N. Driessen. An Interacting Multiple Model Particle Filter. To appear in *IEE Proceedings - Radar, Sonar and Navigation*, 2004.

Some Other Related work:

D.J. Ballantyne, H. Y. Chan and M.A. Kouritzin. A novel branching particle method for tracking. *SPIE Aerosense 2000 proceedings*, volume 4048, pp. 277-287, Orlando FL, 24-28 April 2000.

D.J. Salmond and H. Birch, A Particle Filter for Track-Before-Detect, *In Proc. of the American Control Conference*, June 25-27, 2001, Arlington, VA.

C. Kreucher et al., Multi Target Tracking Using A Particle Representation of The Joint Multi Target Density. *Submitted to IEEE Transactions AES / In Proceedings of SPIE Small Targets Conference, 2003*

Overview

General Particle Filter literature:

As an excellent general book on Particle Filtering with a lot of theory, applications and references:

A. Doucet, N.J. Gordon and N. de Freitas eds. *Sequential Monte Carlo Methods in Practice*, Springer Verlag, New York, 2001.

Conclusions

Specific Conclusions

Every optimal detector can be expressed in terms of PF weights.....Very important result both from a theoretical and practical point of view.

A multi target particle filter for closely spaced targets has been presented for a TBD application. The algorithm can be applied in real time.

Questions/Remarks/Discussions