WORD COMBINATORICS, TILINGS AND DISCRETE GEOMETRY

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The aim of this lecture is to illustrate various connections that exist between word combinatorics, tilings and arithmetic discrete geometry through the discussion of some discretizations of Euclidean objects or of elementary transformations (lines, planes, surfaces, rotations).

Let us illustrate this interaction with the following figure where a piece of a discrete surface in $\mathbb{R}^3$ is depicted, as well as its orthogonal projection on the plane $x+y+z=0$, which can be considered as a piece of a tiling of the plane by three kinds of lozenges, and lastly, its coding as a two-dimensional word over a three-letter alphabet.

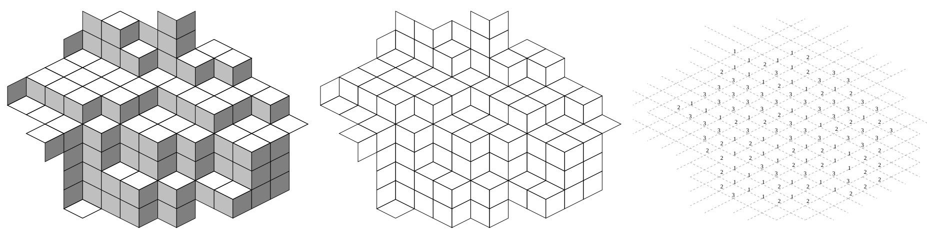


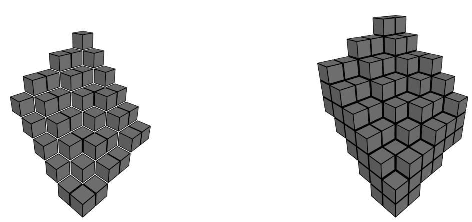
Figure 0.1. From discrete surfaces to multidimensional words via tilings

As an example of a classical dicretization, we will focus on the following notion of arithmetic discrete plane, introduced by Réveillès: the arithmetic discrete plane $\mathfrak{P}(a,b,c,\mu,\omega)$ is defined as

$$\mathfrak{P}(a,b,c,\mu,\omega)=\{(x,y,z)\in\mathbb{Z}^3 \mid 0\leqslant ax+by+cz+\mu<\omega\}.$$

- If $\omega=\max\{|a|,|b|,|c|\}$, then $\mathfrak{P}(a,b,c,\mu,\omega)$ is said naive.
- If $\omega=|a|+|b|+|c|$, then $\mathfrak{P}(a,b,c,\mu,\omega)$ is said standard.

A piece of a naive plane (left) as well as a piece of a standard plane (right) are depicted in the figure below



We will first see how classical techniques in symbolic dynamics applied to some codings of such discretizations allow one to obtain results concerning the enumeration of configurations and their statistical properties. We will then show how natural notions such as morphisms of the free monoid, or flips in tiling theory apply in an efficient way in discrete geometry (generation algorithms, recognition issues).

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